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Risk Measurement, Allocation, and Pricing in Network Schedule Systems

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Risk Measurement, Allocation, and Pricing in Network Schedule Systems

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Construction risk management, controlling the probability and/or severity of potential adverse events so that the consequences are within acceptable limits, is examined relative to network schedule systems. Three analogies are explored for risk related to schedule elasticity and structure, a schedule's participants, and risk's cost; each of which is rooted in related research beyond the bounds of the construction and engineering disciplines.

The inherent risk presented by those operating within the network system, referred to as schedule risk, the likelihood of failing to meet schedule plans and the effect of such failure, is examined with the use of beta (β), the risk correlation of an individual stock to that of the entire market from the Capital Asset Pricing Model (CAPM) of financial portfolio theory, to determine parallels with respect to the inner workings and risks represented by each entity or activity within a schedule to that of the total system or project.

Risk is also viewed through a networks flexibility, herein represented as schedule float, the aggregate time an activity may be extended or delayed without impeding overall outcome, and is explored using the Social Choice voting allocation models and voting power research from the Penrose square root law and the Banzhaf power index.

Float consumption is analyzed via the binomial valuation variation of real options, defined as non-financial options (not derivative-based traded instruments) surrounding tangible assets that creates a future right of choice but not an obligation to pursue a decision, and has its origin in the capital budgeting and financial decision-making process. This forms the basis upon which float is priced.

The long-term goal of this research is to lend insight into schedule volatility, how systematic risk can be quantified, priced, diversified and/or mitigated, and the development of a prediction method for where risk is likely to reside; to provide an alternative to the limitations of 'soft logic' evaluation techniques such as GERT, VERT, and PERT, Graphical, Venture, and Program Evaluation and Review Techniques respectively, and a vehicle for pricing tradable float, all in fulfillment of de la Garza, Vorster and Parvin's unrequited *Total Float Traded as Commodity*.

This dissertation by Richard Clemens Thompson Jr. fulfills the dissertation requirement for the doctoral degree in civil engineering approved by Gunnar Lucko, Ph.D, as Director, and by Lu Sun, Ph.D, and Tongyan Pan, Ph.D. as Readers.

Gunnar Lucko, Ph.D., Director

Lu Sun, Ph.D., Reader

Tongyan Pan, Ph.D., Reader

A schedule defends from chaos and whim. It is a net for catching days. It is a scaffolding on which a worker can stand and labor with both hands at sections of time. A schedule is a mock-up of reason and order – willed, faked, and so brought into being; it is a peace and a haven set into the wreck of time; it is a lifeboat…

Annie Dillard
The Writing Life

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CHAPTER 1

Introduction

1.1 Background

1.1.1 The Construction Industry

The construction industry provides the built infrastructure and physical facilities that support the modern quality of life. It is among the largest industries in the United States and generated nearly $580 billion or approximately 4% of the U.S. gross domestic product (GDP) in 2009 (Bureau of Economic Analysis 2011). In 2010, it employed 5.6 million workers, encompasses 800,000 companies, the majority of which employ less than five individuals (Bureau of Labor Statistics 2011).

The construction industry is considered a seminal industry, because the cost of the fundamental elements in the manufacturing and supply chains affects consumers and impacts the ability of the U.S. to compete in the global marketplace (National Research Council 2007). That is, the distribution and consumption of the goods and services comprising the GDP (every factory, office building, hotel, or power plant constructed) is accounted for within the pricing structure of all goods and services. It also produces long-term unique and complex infrastructure projects, like roads, bridges, refineries, water and sewage treatment plants, and building projects such as houses, apartments, retail establishments, and schools.

Its activities include: (1) construction of new structures including site preparation, (2) renovation, strengthening, and modification of existing structures, and (3) maintenance, repair, and improvements to both old and new facilities (Bureau of Labor Statistics 2011).

Construction is a high-stakes endeavor that faces distinctive challenges with uncertainty appearing from many sources (Miller and Lessard 2001). Each project is unique, is physical, is capital intensive, and is completed within a seasonal environment and tradition-bound atmosphere. Projects require dynamic production systems that use skilled labor, spanning various types of materials and commodities, and include hundreds of different operations (Clough et al. 2005, Ballard and Howell 2003). This creates a highly unpredictable environment where schedule delays and cost overruns are commonplace and inevitable, where risk appears in multiple forms.

1.1.2 Risk and Uncertainty

The construction industry suffers from poor project performance due to risk and uncertainty. With ever-increasingly complex projects, shorter schedules, new procurement methods, scant financing, and the overall dynamic nature of construction projects, the topics of risk quantification, analysis, and mitigation rise to the forefront of importance in today's projects. However, the definitions, communication methods, analysis and mitigation techniques, measurement, and reactions thereto remain inconsistent throughout the construction industry.

The overall assessment and decision of what constitute a risk on a given project is highly subjective and influenced by the past experiences and future perceptions of the project's leadership. Many factors that shape these perceptions, weighing heavily into the quantification process and impacting the path of the project, are not well defined (Tah and Carr 2001). They are the outgrowth of the degree of exposure to said risks during previous experiences and can lead to inappropriate risk characterization when not properly defined or understood.

1.1.2.1 Risk-Related Definitions

Risk, its variations, together with, risk management, risk mitigation, uncertainty, etc. have become ubiquitous in their application and warrant differentiation by definition.

Risk: The effect of uncertainty on objectives, whether positive or negative (ISO 2009), and/or a state of uncertainty where some of the possibilities involve a loss, catastrophe, or other undesirable outcome (Hubbard 2009). Commentary – Risk is the potential that a chosen action or activity (including the choice of inaction) will lead to an undesirable outcome. The notion implies that a choice having an influence on the outcome exists and that risks are future problems that can be avoided or mitigated, rather than current ones that must be immediately addressed. The definition of risk can be parsed for the specifics of the venue or variant. More importantly, as Keynes (1936, p.146) describes the importance of recognizing, acknowledging and being aware of risk and its importance:

> The state of long-term expectations, upon which our decisions are based, does not solely depend ...on the most probable forecast we can make. It also depends on the confidence with which we make this forecast. ...The state of confidence ...is a matter to which practical men always pay the closest and most anxious attention.

Inherent Risk: The probability of loss arising out of circumstances or existing in an environment, in the absence of any action to control or modify the circumstances (WebFinance 2011a). Commentary – Inherent risk in general is the risk found in the environment and in human activities that is part of mere existence, which is impossible to manage, mitigate, transfer, or assign.

Systemic Risk: The risk posed to or faced by an entire system or process, as opposed to risk associated with any one individual, entity, group, or component of a system (WebFinance 2011b). Commentary – Systemic risk is the probability of loss or failure that is common to all members of a class or group or to an entire system and inherent in all workings of a system or process.

Project Risk: A possible event that could endanger the planned course or goals of a project. Commentary – Project risk is the probability that a project will not be completed on schedule and within budget.

Construction Risk: "The exposure to possible economic loss or gain arising from involvement in the construction process" (Erikson 1979, p.3). Commentary – Construction risk is inherent in the work itself; it is the degree of likelihood or probability that an economic loss will occur during a construction project or associated with its post-completion period.

Contractual Risk: The probability of a loss arising from the failure in performing to the expectations of a specific contract or provision contained therein (WebFinance 2011c). Commentary – Contractual risk is primarily caused by a lack of contract clarity, the absence of perfect communication between the parties involved, and the problems of timeliness in contract administration and execution (Erikson 1979).

Risk Management: The identification, assessment, and prioritization of risks followed by coordinated and economical application of resources to minimize, monitor, and control the probability and/or impact of unfortunate events or to maximize the realization of opportunities (Hubbard 2009). Commentary – Risk management is the process of determining the maximum acceptable level of overall risk within a project and from its proposed activity, then using assessment techniques to determine the initial level of risk and, if excessive, developing a strategy to improve upon appropriate individual risks until the collective risk level is reduced to an acceptable level.

Risk Mitigation: A systematic reduction in the extent of exposure to a risk and/or the likelihood of its occurrence. Commentary – Risk mitigation is the act of decreasing the riskiness of a project. In the context of a project, it can be defined as a measure or set of measures a project manager takes to reduce or eliminate the risks associated with the project, its participants or its environment.

Risk Factors: Something that increases the chances that a particular event will occur. Commentary – A risk factor is a variable associated with an increased risk of the occurrence or likelihood of something. Risk factors or determinants (influencing elements) correlate to one another and are not necessarily causal, because correlation does not imply causation.

Uncertainty: The lack of complete certainty. That is, the existence of more than one possibility, where the true outcome/state/result/value is not known (Hubbard 2009). Commentary – Uncertainty is a state of having limited knowledge where it is impossible to exactly describe existing state or future outcome, where more than one possible outcome remains viable and is more general in concept.

The fine distinction between risk and uncertainty and the mutual dependency thereof is best defined by Knight (1921, p.19) in his early twentieth century tome *Risk, Uncertainty, and Profit*. He posits that risk can be measured, while conversely uncertainty cannot:

> [U]ncertainty must be taken in a sense radically distinct from the familiar notion of Risk, from which it has never been properly separated. The term "risk," as loosely used in everyday speech and in economic discussion, really covers two things which, functionally at least, in their causal relations to the phenomena of economic organization, are categorically different... The essential fact is that "risk" means in some cases a quantity susceptible of measurement, while at other times it is something distinctly not of this character; and there are far-reaching and crucial differences in the bearings of the phenomenon depending on which of the two is really present and operating. There are other ambiguities in the term "risk" as well, which will be pointed out; but this is the most important. It will appear that a measurable uncertainty, or "risk" proper, as we shall use the term, is so far different from an unmeasurable [sic] one that it is not in effect an uncertainty at all. We shall accordingly restrict the term "uncertainty" to cases of the non-quantitive type. It is this "true" uncertainty, and not risk, as has been argued, which forms the basis of a valid theory of profit and accounts for the divergence between actual and theoretical competition.

1.1.2.2 Risk Hierarchy

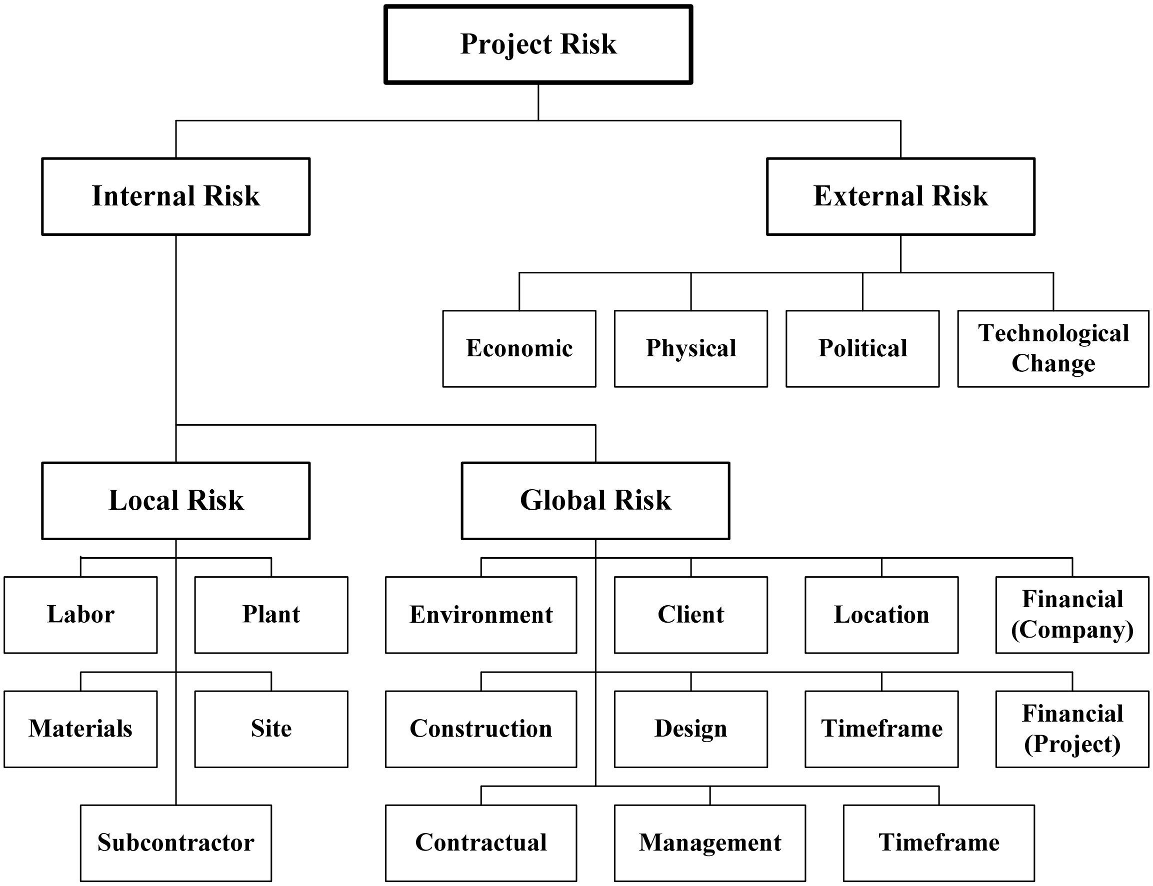
Risk classification is an important element in the risk assessment/risk mitigation continuum of risk management: identify the risk, assess the risk, analyze the risk, handle the risk and monitor the risk. Previous studies depict classification and categorization approaches originating from a multiplicity of perspectives and interdependencies. Risk in general and project risk have been gathered based upon their nature and magnitude, based upon their order of precedence (primary or secondary), based upon their origin and location of their impact, based upon their ability to be mitigated or assigned, and based upon their position in a project's lifecycle or path to completion.

Tah and Carr (2001) present a hierarchy of risks that may be encountered in a construction project and its extended supply chain. Their premise is most *apropos* to construction projects, as risks are separated into two categories based upon the management of internal resources versus those which are relatively uncontrollable and are prevalent in the external environment. They posit that the hierarchical risk breakdown structure (not to be confused with a work breakdown structure) as depicted in Figure 1.1 forms an appropriate model for risk assessment in construction projects.

Fitting within the above hierarchal structure, the sources of uncertainty in construction projects are brought about by nature, by shortcomings in the contractor's internal organization, or by outside influences. In similar fashion, Park (1979) identified twelve major risks that contractors normally face: (1) weather, (2) unexpected job conditions, (3) personnel problems, (4) errors in cost estimating, scheduling, etc.,

(5) delays, (6) financial difficulties, (7) strikes, (8) faulty materials, (9) faulty workmanship, (10) operational problems, (11) inadequate plans or specifications, and (12) disaster.

Figure 1.1: Hierarchical Risk Breakdown Structure (Tah and Carr 2001)



Similar to Tah and Carr's hierarchical risk structure, Cebula and Young (2010) present a taxonomy of risks relative to cyber security with an operational bent. Focusing on information technology assets, they put forward that failure of such assets has a direct negative impact on the business process they support (as can be said of any industry-specific assets), which in turn, can cascade into an overall inability to deliver services (or products) and ultimately hinder the organizational mission.

Their taxonomy attempts to identify and organize the source of operational risks into four classes with subclasses attached thereto (summarized in Table 1.1):

- Actions of People: Action, or lack of action, taken by people either deliberately or accidentally.
- Systems and Technology Failures: Failure of hardware, software, and information systems.
- Failed Internal Processes: Problems in the internal business processes that impact the ability to implement, manage, and sustain cyber security.
- External Events: Issues often outside the control of the organization.

Table 1.1: Taxonomy of Operational Risks (Cebula and Young 2010, p.3)

Actions of People	Systems and Technology Failures	Failed Internal Processes	External Events
Inadvertent Mistakes Errors Omissions **Deliberate** Fraud Sabotage Theft Vandalism **Inaction** Skills Knowledge Guidance Availability	**Hardware** Capacity Performance Maintenance Obsolescence **Software** Compatibility Configuration Management Change Control Security Settings Coding Practices Testing **Systems** Design Specifications Integration Complexity	**Process Design or Execution** Process Flow Process Documentation Roles and Responsibilities Notifications and Alerts Information Flow Escalation of Issues Service Level Agreements Task Hand-off **Process Controls** Status Monitoring Metrics Periodic Review Process Ownership **Supporting Processes** Staffing Funding Training and Development Procurement	**Disasters** Weather Event Fire Flood Earthquake Unrest Pandemic **Legal Issues** Regulatory Compliance Legislation Litigation **Business Issues** Supplier Failure Market Conditions Economic Conditions **Service Dependencies** Utilities Emergency Services Fuel Transportation

It is important to recognize that risks can cascade. That is, a risk in one class or category can trigger a risk in another class or category (whether identified as operational by Cebula and Young or as risks specific to the construction industry by Tah and Carr). Of equal importance is the recognition that the cyber security operational risks while presented as germane to the information technology arena can be extended to most disciplines with little modification. While risks may trigger other risks, risk classifications maintain no dependencies, nor are they mutually independent of each other.

Cebula and Young's four-part categorization can be extended beyond the bounds of cyber security and information technology with little modification. Three of the four classes (Actions of People, Failed Internal Processes, and External Events) require no change or reconsideration for adaptation to other industries, while the fourth (Systems and Technology Failures) remains appropriate, but requires adaptation of its subcategory headings for extension to other industries. For example, the three subcategories of Systems and Technology Failures when adapted to the construction industry, Hardware could become Equipment and not require modification to the Capacity, Performance, Maintenance, and Obsolescence attributes comprising the subcategory. Similarly, Software could become Materials, and Systems could become Plans and Schedules without impacting their respective attributes.

1.1.2.3 Risk Measurement

There are multiple theories for the quantification of risk whether inherent or systemic. Numerous formulae exist for quantifying risk depending upon its origin and the genre to which it belongs. The most widely accepted formula for risk quantification, commonly referred to as the Composite Risk Index, is represented by the product of the rate of occurrence and the impact of the event.

$$\text{Risk} = \text{Probability of Occurrence} \bullet \text{Severity of the Outcome} \qquad \text{[Eq. 1.1]}$$

In its application, the composite index remains entirely subjective, since the probability of risk occurrence is difficult to predict and the impact (severity) of the risk is not easy to estimate. This is due to the severity being generally defined and measured as the ultimate financial loss resulting from the occurrence. In common practice, the probability of occurrence and the severity of outcome may be assessed on a five-point scale, where 1 and 5 represent the minimum and maximums respectively. Thus, the composite index can take values ranging from 1 to 25, and can be stratified into sub-ranges representing an overall risk assessment of low, medium, or high; where the sub-ranges are grouped into three approximately equal sets from 1 to 8, 9 to 17, and 18 to 25 respectively.

1.1.2.4 Risk Management / Mitigation

In practice there are five ways to manage or mitigate risk: Assume it, don't assume it, abate it, allocate it, or transfer it (Abramowitz 2009). One is not preferred over the others, but is dependent upon the specific risk, the juncture in the lifecycle of the project at which it occurs, the capabilities and competencies of those facing the risk, the magnitude of the risk, and/or the potential for successful outcome.

Assume the Risk: Take responsibility for the risk and its cure. This requires the confidence that the requisite skills are present or available through others to remedy the situation.

Don't Assume the Risk: Walk away from the risk and leave it to others. This presupposes the ability to do so, or sufficient time in advance of the project, work and risk to properly establish contractual parameters to exclude responsibility.

Abate the Risk: Reduce the amount, degree, or intensity of the risk or the exposure to the risk by obtaining superior knowledge and know-how.

Allocate the Risk: Make the risk the responsibility of others more capable of addressing it. Assign the responsibilities and provide the empowerment necessary to overcome the risk while maintaining oversight.

Transfer the Risk: The insurance and indemnity approach to limiting exposure to risk: pay someone else to take on the risk.

These five approaches represent a loose division of options to manage and/or mitigate risk. However, not all risks are clear-cut and simple to identify and compartmentalize this easily, nor can most risks be solved through a single means.

1.1.2.5 Risk Allocation

Risk allocation is the technique whereby specific individual risks are distributed among a project's participants. Its premise is that doing so will result in multi-beneficial outcomes to all parties involved (Vega 1997). The chances of successful project completion are purported to decrease significantly should only a single entity or party to a contract (typically the general contractor) bear all of the project's risk. In this sense, it is paramount that the basic concessions made by the parties to the project in response to the assumption of risk are fair and acceptable to all involved. Such distribution across the project participants allows them to manage uncertainty more efficiently (Peña-Mora et al. 2003).

1.1.3 Construction Project Management

Construction managers are tasked with planning and controlling all of the technical and non-technical aspects of their projects. The latter's project management "dimensions" are time and cost. They are highly interrelated (Kerzner 2003) under the desired scope of the project. Additional aspects are resources (e.g. labor and equipment), technical means and methods, communication, permits, and quality. Coordinating them is subject to various constraints and the overarching concern for safety.

Construction projects are temporary endeavors with a specific purpose that are created by expending resources (Project Management Institute 2008), and in this research refer to facilities or buildings. Project management seeks to deliver projects safely, on schedule, within budget, and to the desired specifications, based on the best data available, conscious of risks, and prepared to mitigate them.

Project planning is the process to develop the approach to fulfill the project expectations and predefined objectives. It involves choosing among multiple courses of action and is an early representation of decision-making and risk analysis. Jaafari (1984) and Glavinich (1994) identify project planning as having seven objectives:

1. To balance uncertainty and modification.
2. To acquire a thorough understanding of project objectives and focus attention on predefined objectives.
3. To achieve economical operations and formulate strategies for achieving project objectives using available resources.
4. To facilitate control by developing a framework for monitoring and directing the project.
5. To allocate contractual responsibilities and provide clear lines of communication.
6. To coordinate contributions from various groups, during the engineering phase.
7. To mitigate and resolve delay and change order disputes on a predefined, quantifiable and equitable basis.

Faniran, Love and Li (1999) concluded that an optimal amount of project planning should be invested at the project outset to support the ultimate goal of construction projects: completing the project on time and within budget. Beyond a certain point, continued or increasing levels of planning increase the probability of poor project performance and increased overhead costs, as the level of specificity and schedule granularity becomes too fine, unachievable, and provides little return for the effort invested.

1.1.4 Project Schedules

Construction project complexity and cost intensity necessitate the development and monitoring of schedules. Uncertainty and the circumstances of risk compound time constraints and their impact upon a project (Mulholland and Christian 1999), thereby rendering the project schedule one of the most important elements in planning and executing construction projects. Most projects are deadline driven, timeliness is legally construed as 'of the essence,' and the conditions of performance are always non-predictive.

At the outset of most projects, schedule overruns are not seen as probable, for which planning is not robust, even by experienced project managers (Ambani 2004). Unforeseen events may arise and adversely impact a project's schedule. More often, schedules are overrun because of commonplace occurrences such as design problems and trade disputes which are neither entirely unforeseeable nor unpredictable. However, their likelihood and magnitude of impact are difficult to predict with any degree of certainty due to the individual characteristics presented by individual projects (Mulholland and Christian 1999). Consequently, for projects with any amount of uncertainty, network-based planning processes have been proven inadequate for estimating a realistic project performance time (Laufer and Howell 1993).

Failure to fulfill schedule milestones and completion commitments usually results in accretive costs in multiple forms, including some punitive in nature. Owners and contractors face extended operational costs (costs associated with time and labor) to complete the project, as well as lost income associated with beneficial use of the facility and risk liquidated damages as financial penalty for finishing beyond the contractual completion date (Schumacher 1996, Householder and Rutland 1990), as well as delaying subsequent projects.

To gauge the impact and monitor the resolution of adverse occurrences during project completion, scheduling is unquestionably in the forefront. Its primary objective is to promote orderly construction operations and assure that sufficient levels of quality are provided when deadlines are challenged. Beyond this, accurate and updated project schedules are an integral element for the resolution of any dispute that may arise from unmet schedule obligations, which may in and of itself result in further schedule delays.

To analyze the source, responsibility, and ultimate liability for unfulfilled schedule obligations and negotiate a fair and timely settlement deemed as beneficial to *all* parties to a construction project by Kraiem and Diekmann (1987), the construction industry uses many scheduling practices and techniques, but is challenged with prospectively identifying to any level of accuracy the scope or magnitude of change-caused disruption at the activity level (Finke 1998).

1.1.5 Schedule Impacts

Ideally, from the perspective of project scheduling, the goal of every project is that its activities follow the sequence of early starts and early finishes. This negates the requirement for time adjustments and the need for flexibility within the planned versus actual sequence of events. Float, in general, would not be needed nor consumed; interim, milestone and final deadlines and completion dates would be met; no claims would be filed; liquidated and consequential damages would not be imposed; and more importantly schedule time extensions would not be sought.

However, in the uncertain construction environment, this scenario does not materialize, and the schedule is impacted in a negative manner. Schedule delays (schedule impacts – events that prevent the timely completion of the work within the contractually specified time period (Wickwire et al. 2003)) materialize from a variety of sources and take on multiple forms. Arcuri and Hildreth (2007), together with definitions from Bartholomew (2002), Bramble et al. (1990), and Parvin (1993), identify and define the foremost project impacts as:

Delay: The lack of performance or the extension of time required to complete a project that results from unexpected events; may be caused by the contractor the owner, third parties, or by unanticipated natural or artificial site conditions.

Disruption: The lost productivity that results from interruptions in the planned sequence of operations.

Change: Any type of work that deviates from the original contract, or from the scope of work or plan of action reasonably anticipated under the contract.

Suspension: A written directive by the owner to stop all work on the project, either because the contractor has failed to perform in accordance with contract documents, or at the owner's convenience.

Termination: The cessation or cancellation, in whole or in part, of work for the convenience of, or at the option of, the owner, or due to failure to perform (default) in accordance with the terms of the contract.

Arising from the establishment and maintenance of a project schedule (described below with respect to the ubiquitous Critical Path Method) is the analytical methodology deemed Schedule Impact Analysis; a method for understanding, quantifying, and apportioning schedule delays. It's formally defined in two parts: (1) schedule impact, the potential effect of a delay or the change to a project schedule that may be in the form of delay or change to the project completion date, a delay or change in the project sequence, or an event that requires consumption of float, and (2) schedule impact analysis, the process of quantifying and apportioning the effect of delays or changes to the project schedule (Arcuri and Hildreth 2007).

Schedule impact analysis incorporates multiple techniques to determine the total impact of the delay and isolate its effect on the entire project. Several are retrospective in their application and others concurrent or contemporaneous techniques. The first two are considered illegitimate, as they make conclusions on the effect of delays without considering any project logic (they don't incorporate the use of a CPM schedule) (Henschel and Hildreth 2007):

1. Global Impact Approach (Illegitimate)
2. Net Impact Approach (Illegitimate)
3. Adjusted As-Planned CPM Approach (Retrospective): After the fact inserting delays into the as-planned schedule to quantify the global impact.
4. Adjusted As-Built CPM Approach (Retrospective): After the fact insertion of delays into the as-built schedule to show the "critical path" and quantify the global impact.
5. Collapsed As-Built (But-for) Schedule Approach (Retrospective): After the fact delays are subtracted from the as-built schedule to quantify the global impact.

6. Impacted Updated CPM Approach (Retrospective): After the fact inserting delays into an updated as-planned schedule to quantify the localized impact. Note: This technique may also be employed contemporaneously.
7. Modification Impact Analysis Approach (Contemporaneous): At time of modification the schedule is updated and the delay inserted to quantify the singular impact.
8. Time/Schedule Impact Analysis Approach (Contemporaneous): Recreate time of modification by using updated schedule and inserting the delay to quantify the singular impact. (Bramble et al. 1990)

Common to each of these techniques and as described by the U.S. Army Corps of Engineers' *Modification Impact Evaluation Guide* (1979) is process followed to implement schedule impact analysis, a simultaneous at best but characteristically posthumous approach to addressing the uncertainties encountered during a construction project:

1. Determine the actual status of the job when the delay occurred.
2. Analyze the scope of the modification to determine which activities will be directly affected, and modify schedule to accommodate affected activities.
3. Use revised schedule to determine new critical path and completion date, which may issue a time extension and/or damages.

1.1.6 The Critical Path Method (CPM)

Traditional project planning is strongly centered on the discrete scheduling of time. Scheduling as currently practiced originated in 1956 when Kelley and Walker (1989, 1959) planned a factory with an early UNIVAC computer. The critical path method facilitates forecasting to an accepted level of accuracy, the allocation of time to complete a project, and depiction of the inter-relationship of the individual tasks into which the overall project is divided (known as the work breakdown structure). A CPM schedule is designed to advise the parties involved about the relative importance of performing certain activities within the project completion parameters, the predetermined duration and sequence: All activities in a network schedule are linked via their starts (*S*) or finishes (*F*) with either *F*-*S*, *S*-*S*, *F*-*F*, or *S*-*F* sequence that may carry a lag duration. CPM scheduling adapts linear programming (Dantzig 1955) to solve a system of equations of the type "*start plus duration equals finish*" ($S + D = F$).

When activities are knitted together to form an overall schedule, variations in activity durations compensate each other, resulting in a reasonably accurate network schedule system. Upon determining the applicable classification for an activity (predecessor, successor, or concurrent / independent, and the forward and backward passes), the early start and early finish times and late finish and late start respectively, the critical path, the longest uninterrupted path from beginning to end of the project, can be determined. The critical activities (those along the critical path), become apparent. Critical activities (those without float or slack time between subsequent activities), and non-critical activities (those not on the critical path with available float), are determined.

However, the degree of criticality remains arguable as the spectrum of float varies greatly, in-theory zero to infinite. The potential for sub-critical activities not on the critical path but with little float so as to render their duration and flexibility to absorb delays the same as a critical activity for which a second or near critical path may exist, requiring identification and specific consideration in planning efforts.

CPM facilitates management by exception (Kim 2003), which means that the activities that are critical or near critical can be forecast and become known to a project's managers. This facilitates the realization of the activities most in need attention, presupposes mitigation, and is one of the principal advantages of the critical path method. It is ubiquitous across the industry (Galloway 2006).

1.1.7 Total Float

The use of network schedule system logic and characteristics makes possible the development and/or calculation of other information, such as float, the flexibility within a project's schedule. In general, float within CPM schedules is defined as the lag time, slack, or buffer that the start of an activity can be delayed. Among the multiple definitions of float are two preeminent forms: free float and total float. Total float is the amount of time that an activity can be delayed without affecting/delaying the overall project completion date, and free float is the amount of time an individual activity can be delayed without having to reschedule any other activity in the project. Consuming total float does not impact the ultimate completion of the project (de la Garza et al. 1991) and is therefore a key issue in the management and completion strategy of a project.

The consumption of float is an important means to reduce the impact of risk on projects. It is considered *highly valuable* but a *vanishing commodity* (Wickwire et al. 2003) and is generally governed by tradition and contract language. It belongs to no single project entity and is expended on a first-come, first-served basis (Pasiphol 1994). Total float is shared along a path, and all successor activities may claim it for their own benefit. It is of primary use in project management to reduce schedule impacts, as it can compensate for delayed activities and subsequent lost time. However, when two or more parties on the same schedule path share concurrent float time, problems arise in relation to float consumption and ownership. This behavior links the still-unanswered question, *"Who owns float?"* (Person 1991) with the equally important *"Who should own risk?"*

1.1.8 Float Allocation – Current Understanding

Float, a vanishing valuable commodity, initially belongs to no single entity (Wickwire et al. 1991). It is claimed on a first-come-first-serve basis, rather than maximizing its real potential as a shared benefit for all participants. Al-Gahtani (2009) identifies numerous approaches to the ownership and consumption of float: owner ownership, contractor ownership, the project approach (joint ownership), the bar approach, equal proportion, commodity approach (trading approach of de la Garza et al. (1991)), contract risk, path distribution, day-by-day, and total risk. Overarching this list can be found three primary taxonomies: (1) owner entitlement, (2) contractor entitlement, and (3) joint or project ownership or control based upon a distributive mechanism or set of determining factors, with joint or project ownership being the focus of this research.

1.1.8.1 Owner Entitlement to Total Float

Under the owner entitlement to total float precept, the owner should have the right to appropriate the total float for its specific use. The motivation for this approach is based upon theory that it is the owner that pays for the project, and in particular forms of contracts (cost-plus variations) bears the greatest financial risk of all parties. Thus, the owner should be able to use the total float to manipulate the float to its best advantage and reduce costs as much as possible. This approach favors the owner, because it provides flexibility for incorporating changes to the project without time extensions, thereby ensuring timely completion of the project. Should total float ownership not be addressed in the construction contract, the preponderance of legal decisions has given total float ownership to the owner (Callahan et al. 1992). This ownership allocation causes dissatisfaction to contractors, because they lose flexibility in managing (maximizing) their resources within projects and across concurrent projects (Pasiphol 1994).

1.1.8.2 Contractor Entitlement to Total Float

Under the contractor entitlement to total float precept, contractor should have the right to appropriate the total float for its specific use. The motivation for this approach is based upon theory that it is the contractor's responsibility for the means and methods of construction (the planning scheduling allocating the resources and equipment necessary for the project) to complete it in a timely manner and fulfill its contractual obligation. Given this overall responsibility, the contractor should be able to control the sequences and durations of the various project activities and resources to maximize its potential. This approach favors the contractor, because it allows the optimization of operations across all work and the avoidance of additional and unnecessary costs (Person 1991).

Case law exists for the contractor ownership of total float. It is based in part on the supposition that it is the contractor who develops and maintains the construction schedule of which total float is an element. It represents a cushion of time to which the contractor is entitled irrespective of the overall issue of float ownership (Person 1991). Similarly, the U.S. General Services Administration asserts that the contractor is entitled to use float as a resource within its responsibilities for planning and scheduling the work.

1.1.8.3 Joint or Project Entitlement to Total Float

Under the joint or project entitlement to total float precept, float belongs to all participants in the project. Stated conversely, total float under this entitlement variation belongs to no singular or specific entity. All project participants should have the right to appropriate the total float for their specific use. The motivation for this approach is based upon theory that when total float is first needed, it is consumed. Simply put, whoever gets to total float first can reap its benefits. Without sufficient float as project activities ensue, those non-critical activities that perform activities later in the project must perform their work as critical activities (Pasiphol 1994).

This approach favors early participants, as those with late-stage project activities may be liable for damages should any schedule delays occur that result in an extension of time beyond the remaining total float (if any) and thereby not fulfill their contractual obligation. Building upon this supposition, contractors may create unrealistic schedules and inaccurate updates regarding construction progress to report faulty amounts of total float or to show owner-caused delays along the project's critical path (Prateapusanond 2003).

This approach appears to be successful at reducing ownership conflicts solely because of its dependence upon the logic of the project network schedule. However, in reality problems arise because of the propensity for the aforementioned CPM network distortions. Therefore, project schedules may not reflect what the real logic is and how the project will actually be executed.

1.1.8.4 Consumption of Total Float

Irrespective of entitlement approach, all project participants are likely to consume float for their benefit. Owners consume total float when they (1) fail to make decisions and/or perform their required duties in a timely manner, (2) make changes to the scope of work, (3) interfere with the contractor's orderly completion of the work, (4) include 'no damages for delay' or 'owner control of float' provisions within contracts, or (5) their agents and representatives (architect, engineer, construction manager, or other prime contractors) likewise fail to make decisions and/or perform their duties and responsibilities in a timely manner (Vezina 1991, Householder and Rutland 1990).

Contractors consume total float when they: (1) fail to perform their work properly – e.g. fail to coordinate the work of subcontractors, poorly manage resources and time, fail to include subcontractor's schedules into the baseline project schedule, etc., (2) lack a realistic schedule, (3) engage in 'schedule games,' e.g. use target or conditional dates, use preferential sequencing, use artificial activity durations, inaccurately update the schedule, change project and schedule history, or (4) through the actions or inactions of their subcontractors or suppliers (Hulett 1995, Vezina 1991, Householder and Rutland 1990).

In reality, construction activities may or may not finish on schedule. Individual activities may finish ahead of the planned completion date, or they may finish behind the scheduled date. As total float is consumed through the course of a schedule and the uncertainty of time, they become more difficult to manage and control (Fink 2000). As a result, through the increase in the rate of total float consumption, the risk of schedule overruns and cost increase (Gong 1997).

1.1.8.5 Total Float Traded as Commodity

Under the total float traded as commodity precept, float should be shared or traded among the project's participants based on activity cost. That is, the project participants should have the right to trade, to buy and sell total float, among themselves. The motivation for this approach s based upon theory that since contractors apply total float to reduce fluctuation in project resources over a defined period of time, it should be the contractor's property with a trade-in value. The commoditization of total float comes out of the belief that the cost of an activity during the early part of the schedule versus the later part may not be the same and should be offset to the participants by monetary compensation.

In their seminal work *Total Float Traded as Commodity*, de la Garza, Vorster, and Parvin (1991, p.719) present this approach and state:

> Because both owners and contractors can gain or lose if unforeseen conditions effect the project scope or project schedule, contractors not only have the right to administer and use total float but also the obligation to trade it. Thus, to maintain equilibrium from agreed-on-risk-sharing expectations, flexible time taken away from the schedules needs to be replaced with monetary contingencies.

This presupposes that the early total float of the project becomes more valuable because of the downstream impacts on succeeding activities. De la Garza et al. estimated the trade-in value or exchange rate of total float based upon the estimated costs of a late finish versus an early finish. That concept this allows for the value of total float during one period of a project to be more valuable than that of another period, depending upon the status of the project and the criticality of float consumption.

$$\text{Total Float Trade-in Value} = \frac{\text{Late Finish Cost} - \text{Early Finish Cost}}{\text{Total Float}} \quad \text{[Eq. 1.2]}$$

1.1.8.6 Entitlement Ambiguity

The wealth of float ownership and allocation approaches results in an ambiguous situation, thereby becoming a major source of delays leading to litigation (Prateapusanond 2003). De la Garza et al. (1991) cite disputes over the ownership of total float as being at the core of most delay claims. In practice, float ownership is rarely allocated to the parties at the outset of the project. When delays occur, disputes and conflicts arise followed by all parties trying to assert their rights to use the available float in whole or in part to minimize their delay responsibilities. Understanding the concept of float time, its behavior, and its consumption are important for all project participants. Clearly allocating and specifying the amount of float time to all contractually responsible parties are key factors in avoiding conflicts (Pasiphol 1994).

Total float is used to compensate for delay problems and lost time as they occur. However, with parties on the same schedule path sharing the same float time, ownership and/or entitlement issues arise surrounding how it can be fairly distributed and consumed. Callahan and Hohns (1998, p.135) state that:

> Courts and Boards that have considered the ownership of float have not reached similar conclusions. Contributing to the inconsistent decisions is the conflicts between two common provisions in construction contracts. First, that the risk of construction lies with the contractor, tends to support owner claims to float… Second, that the contractor is responsible for the means methods and techniques of construction, tends to support the contractor’s claims to float.

This lends credibility to the need for a comprehensive and systematic method of total float allocation via an agreement prior to the start of a project, that is, before delays can surface and disputes arise.

1.1.9 The Pre-Allocation of Total Float

The concept of allocating total float and its ownership has been employed in an informal manner in construction scheduling and remains a source of dispute and ambiguity among participants. Several attempts have been made to define methods by which total float may be controlled by its allocation, designation, reservation, or exchange in advance of or during a project. Four methods have been used or introduced to either allocate or control float, including (1) direct allocation of float to individual activities along a path of activities, (2) calculating and using ‘safe’ float, (3) constructing and using float clauses within contracts, and (4) trading total float as commodity.

1.1.9.1 Allocation of Float to Individual Activities

Two variations for allocating float to individual activities along a network schedule path exist. The first is percent-based using the individual activity duration ratio of the total activity path as the relative percentage for distributing the total float of the specific activity path (Wickwire et al. 1991):

$$\text{Allocated Float} = \frac{\text{Activity Duration}}{\text{Total Duration of Activity Path}} \cdot \text{Total Float on Path} \qquad \text{[Eq. 1.3]}$$

The second concept for distributing total float to each participant furthers the above method by allocating float along the same activity path not by percentage, but rather by quantitative and qualitative criteria. Pasiphol and Popescu (1995) and Pasiphol (1994) identify three quantitative criteria to be used in allocations: (1) Uniform Distribution – equal allocation to each activity, (2) Activity Duration – proportional allocation based on time, and (3) Activity Direct Cost – proportional allocation based on direct cost of the work.

Their qualitative criteria classification surrounds the numerous and non-numeric factors that can cause delays and allocating more float to where these items are likely to reside and/or occur. Possible qualitative criteria include (Prateapusanond 2004):

1. *Activity Resource Demand:* Allocation of more total float to the activity that requires more resources, e.g. special materials or equipment.
2. *Labor Strike Prone:* Allocation of more total float to the activity that is more prone to the effects of a strike.
3. *Late Material Delivery:* Allocation of more total float to the activity that has a higher risk of late material delivery.
4. *Type of Work:* Allocation of more total float to the activity requiring highly skilled labor for completion or to the activity requiring stringent quality control.

5. *Insufficient Drawings and Specifications:* Allocation of more total float to the activity possessing the most complex drawings.
6. *Environmental Permission:* Allocation of more total float to the activity with environmental considerations, e.g. environmental permit requirements, disposal of hazardous materials and/or waste.

Application of this method of total float allocation is easily achieved in practice, but remains subjective in its qualitative form and disconnected to the occurrence of and potential for risk in its quantitative form.

1.1.9.2 Calculating and Using 'Safe' Float

Gong and Rowings (1995) introduced the use of 'safe' float, the amount of float that can be used to safely reduce the risk of project delays to non-critical activities. Updated by Gong in 1997, this method measures the joint influence on float use together with the uncertainty of non-critical activities on the project duration. They identified this as the 'combined influence,' and suggest a calculation of safe float for use in 'risk-analysis-oriented' network scheduling. This requires measurement of the joint influence of two activities at their juncture and merger, i.e. the combination of activity A with activity B into a new singular activity C and a shortened expected duration.

The concept involves two basis elements: A back-forward uncertainty estimation process (the influence of non-critical activities at each merge event in a project schedule (Gong and Rugsted 1993)) and the development of a safe float range defined as "the amount of float use in noncritical activities that does not lead to a disturbance in the total project time" (Gong and Rowings 1995, p.189). It suggests that as long as all parties consume float in the recommended safe float range, the risk of project delay caused by total float consumption is minimized, and total float ownership does not become an issue.

This method is not used in practice. While it shows that the risk of schedule overruns and the associated costs can increase if total float is consumed beyond a certain point, the decision to pay an increased price associated with this approach at the outset may not outweigh the future cost of an incurred delay. One reason for this is that the defined safe range of float to use is subjective to the project manager or scheduler's expected magnitude in the changes in time when activities or events are merged. Secondly, its application may prove to be far too complex, and the risks covered by the safe float range may not materialize.

1.1.9.3 Contractual Float Clauses

The most prevalent float management and allocation method is by inserting float ownership clauses and/or other scheduling clauses into the construction documents (specifications) during the design phase and subsequent negations in advance of executing a construction contract. Numerous studies concluded that good and fair project scheduling can be accomplished if during the design phase scheduling specifications are well prepared and well written (Sweet 1999, Hartman, et al. 1997, Zack 1996, Ibbs and Ashley 1986, and Ashley and Mathews 1984).

Typically, convention is in United States that float belongs exclusively to the contractor (Zack 1996), as this is where schedule risk and responsibility reside. To overcome this presupposition, clauses may be included within construction contracts to address float ownership in the forms of:

1. *Joint Ownership or Sharing:* Used mostly by the U.S. Government to reverse the perspective that the contractor owns the float (Person 1991). Such a clause simply states that float is a jointly-owned resource and is consumed on a first-come, first-served basis. Building upon this, Prateapusanond (2004) presented a vehicle for pre-allocation of total float in an equitable manner between owner and contractor, thereby giving equal rights to the float of non-critical activity paths.
2. *No Damages for Delay:* Used as an enhancement of the joint ownership precept, such a clause in its purest form limits an owner's exposure to delay claims by stating that no time extensions will be granted nor will any delay damages be paid resulting from a schedule delay. Other variations exist that pull back the exhaustive nature of these clauses to make them more palatable and in-line with public policy. They seek to limit the no damages for delay provision to extensions beyond the current agreed upon completion date that are expressly caused by the owner or their agent. Many jurisdictions prohibit these clauses in public contracts, and a few have even banned them outright by deeming such clauses wholly "void and unenforceable."
3. *Nonsequestering of Float:* Used to curb the practice whereby the contract can sequester or take control of float in a project schedule by using preferential logics, artificial activity durations, or constraints. Such a clause eliminates the possibility for engaging in these practices by merely giving the owner authority to review and reject the schedule if float is sequestered.

Prateapusanond (2004) identifies four contractual clauses necessary in fulfillment of the pre-allocation of total float: (1) float definition, necessary to define the meaning of total float, (2) pre-allocation of float, necessary to define the concept of total float pre-allocation and its application within the project, (3) no damage for non-critical delay, necessary to address delays to non-critical activities whose accumulation does not negatively impact the project completion date, and (4) formulas, necessary to properly quantify the two parties' responsibilities.

Prateapusanond (2004, p.107) also supposed that total float should be allocated between owner and contractor equally:

> The proposed concept for managing "total float" seeks to alleviate such potential problems [overconsumption] by pre-allocating a set amount of total float on the same non-critical path of activities to the two contractual parties, the owner and the contractor. This research further proposes that such pre-allocation of total float must be clearly and expressly stated by specific clauses in the prime contract.
>
> The pre-allocation of total float can range from 0-100 or 100-0. However, in order to achieve equity, this research recommends 50-50 allocation. This policy functions to give the owner and the contractor equal rights to the total float, as shown in the project network schedule—stated another way, the owner and the contractor each owns one-half of the total float available on any non-critical path activity of the project.

1.1.9.4 Trading Total Float as a Commodity

As mentioned above, de la Garza, Vorster, and Parvin introduced a concept that treats total float as a commodity that is beneficial to both owners and contractors. Its primary assertion is that both owners and contractors can gain or lose if unforeseen conditions affect the project scope or schedule. Therefore, contractors not only have the right to administer and use total float but also the obligation to trade it on demand (de la Garza et al. 1991). By logical extension, total float taken away from the schedule needs to be replaced with either incentive or monetary contingencies (under the aforementioned calculation) and is treated as any other resource.

Prateapusanond (2004) indentifies the complications and shortcomings of this approach as attributed to the dynamic nature of the schedule whereby activity durations and logic of the CPM schedule can be adjusted during the construction period. The total float will, of course, be changed as well, rendering implementation of this concept both difficult and impractical.

Here resides the launching point of this research: the lack of a comprehensive approach for the identification, allocation, and valuation of risk found within network schedules as exhibited by the consumption of float.

1.1.10 Alternative Theories and Practices

Beyond the traditional methods for managing resources and time via network schedule systems, e.g. the critical path method (CPM), and the soft logic methods evaluation methods of Graphical (GERT), Venture (VERT), and Program (PERT) Evaluation and Review Techniques respectively, two other methods for optimizing schedule performance have advanced: Critical Chain Project Management (CCPM) developed by Goldratt (1997) originates in the *Theory of Constraints*; and Lean Construction philosophy, an extension of lean manufacturing principles and practices stemming from the Toyota Production System. These methods are applied to the construction process with a concern for continuous improvement and the abandonment of the time-cost-quality tradeoff paradigm.

1.1.10.1 Critical Chain Project Management (CCPM)

The concept of Critical Chain Project Management was introduced in *Critical Chain* (Goldratt 1997). Its premise is to keep resources level but require flexibility in start times and the ability to quickly switch between tasks and "task chains" to keep the entire or overall project on schedule.

The critical chain is the sequence of both precedence- and resource-dependent elements that prevents a project from being completed in a shorter time. By definition, when resources are constantly available in unlimited quantities, a project's critical chain is identical to its critical path. Where the critical chain method differs from the critical path method is found in features such as: (1) The use of resource dependencies as the project constraint, (2) the lack of a search for (need of) an optimal solution, (3) the identification, insertion, and monitoring of project, feeding and/or resource buffers, and (4) encouraging reporting early completion of activities (Leach 1999). Of primary importance to the critical chain method is monitoring the consumption rate of the buffers (Steyn 2002), rather than monitoring individual task performance to an overall project schedule as is practiced by the critical path method.

In practice, CCPM seeks to aggregate all of the large amounts of 'safety time' that are added to tasks within a project (extended task completion timeframes and/or float of various types as found in CPM) to form buffers to protect completion dates and to avoid wasting it (the safety time) through poor performance.

Raz and Barnes (2001) trace the creation of a CCPM project plan in much the same process as CPM: CCPM works the conceived plan backwards from a predetermined completion date and establishes each task's starting date as late as possible. This establishes its duration. Then CCPM establishes other durations, a 'best guess duration' at an aggressive 50% probability of occurrence and a safe duration at a significantly higher probability for achievement in the 90% to 95% range (and a correspondingly longer duration).

CCPM's 'best guess' and 'safe' durations are likened to the Project Evaluation and Review Technique (PERT) concept and its component part definitions and duration formula (Malcolm et al. 1959):

- *Optimistic time* (O): The minimum possible time required to accomplish a task, assuming everything proceeds better than is normally expected
- *Pessimistic time* (P): The maximum possible time required to accomplish a task, assuming everything goes wrong (but excluding major catastrophes).
- *Most likely time* (M): The best estimate of the time required to accomplish a task, assuming everything proceeds as normal.
- *Expected time* (T_E): The best estimate of the time required to accomplish a task, accounting for the fact that things do not always proceed as normal (the implication being that the expected time is the average time the task would require if the task were repeated numerous times over an extended period of time), where,

$$T_E = (O + 4M + P) / 6 \qquad \text{[Eq. 1.4]}$$

Resources and buffers are then assigned to the tasks at the 50% probability expectation (recognizing that it is more likely that individual tasks will take more time than initially allotted) and that at the 50% level, half of the tasks will take more time and half will take less time. The extra duration of each task on the critical chain, the difference between the safe durations and the 50% durations, is aggregated in a buffer at the end of the project (Stratton 2009). It is this date (duration), at the end of the aggregate project buffer, that is established as the ultimate project completion date.

At a conceptual level, CCPM differs from the CPM in that it seeks to aggregate as much surplus time, safety time (the CCPM term) or float (CPM term), at the end of project activities, not throughout the schedule assigned to or associated with individual activities. Conversely, a CPM schedule may be interlaced with hidden schedule buffers in the form of exaggerated individual activity durations.

1.1.10.2 Lean Construction Principles

The term Lean Construction was coined by the International Group for Lean Construction in its first meeting in 1993 (Gleeson and Townsend 2007). It is conceptually defined as a "way to design production systems to minimize waste of materials, time, and effort in order to generate the maximum possible amount of value" (Koskela et al. 2002, p.211). It is predicated upon an analysis of project plan failures that indicates that "normally only about 50% of the tasks on weekly work plans are completed by the end of the plan week' and that constructors could mitigate most of the problems through 'active management of variability, starting with the structuring of the project (temporary production system) and continuing through its operation and improvement.'" (Ballard and Howell 2003b, p.2).

Lean Construction is inextricably tied to Lean Production, "the practice of better meeting customer needs while using less of everything" and is particularly well-suited for complex, uncertain and quick projects (Howell 1999). Lean Production (or Lean Manufacturing as it is also known) has its origin with Toyota and their lead engineer, Taiichi Ohno, in the late 1940s, but was first coined by Krafcik in *Triumph of the Lean Production System* (Krafcik 1988). It continued to develop over the next quarter century. Ohno's internal research team worked to shift attention from the entirety of the production system to focus on individual worker productivity and mass production with machines, with the overall objectives of the Toyota Production System being to design out overburden and inconsistency, and to eliminate waste. Lean Production continues to evolve today, but remains true to Ohno's initial premise of delivering a custom product instantly while maintaining no intermediate inventories. It is characterized by the following precepts (Howell 1999):

- Identify and deliver value to the customer; eliminate anything that does not add value.
- Organize production as a continuous flow.
- Perfect the product and create reliable flow through stopping the line, pulling inventory, and distributing information and decision making.
- Pursue perfection: deliver on order a product meeting the customer requirements with nothing in inventory.

Lean Construction differs from Lean Production in that construction is project-based, whereas manufacturing is entirely process-based. Construction projects remain site specific, are most often singular in occurrence, and are constructed in a stationary manner. Manufacturing utilizes a stationary production facility based upon the movement of product components along a fixed, predetermined path or process. In construction workers move and the product is stationary; in manufacturing, the product moves between stationary workers.

Overall, Lean Construction is concerned with the holistic pursuit of concurrent and continuous improvements in all dimensions of the built and natural environment: design, construction, activation, maintenance, salvaging, and recycling (Abdelhamid 2007).

1.2 Research Objectives

Risk is ever-present in construction projects irrespective of their size. Managers must identify the types, causes, and effects of risk and choose strategies for its avoidance or mitigation (should it occur). Construction and project managers must have an array of contingencies and options to transfer risk to participants who are equipped, willing, and/or paid to handle the risk, all of which is predicated upon the ability to measure, assign and price the cadre of project risks.

The primary purpose of this research is to seek proven concepts from the areas of finance and social decision-making and through extension and adaptation transfer this knowledge to network scheduling systems to show how systematic risk can be quantified, priced, diversified and/or mitigated using accepted and seemingly unrelated concepts in fulfillment of the *Total Float Traded as Commodity* notion posited by de la Garza, Vorster, and Parvin (1991) 20 years ago.

To accomplish this goal, a proper foundation for the need or justification for the research must be established, the limitations to its scope and the assumptions made warrant identification, any previously unanswered research questions require enumeration and consideration, the research questions / objectives to be addressed need definition, and the research process requires delineation.

1.2.1 Justification for the Research

The consumption of float (i.e. the flexibility within a project schedule) is a most important means to reduce the impact of risk on projects. Float is an element arising from Critical Path Method (CPM) schedules and is considered highly valuable, but a vanishing commodity (Wickwire et al., 2003). It is typically governed by tradition and contract language, belongs to no single project entity, and is expended on a first-come, first-served basis (Pasiphol 1994). Herein lies the foundation of the problem to be considered by this research *no approach exists that provides a theoretical framework for the fair quantification, allocation and valuation of float to mitigate risk in projects governed by network schedule systems*.

This need, initially identified (below) by de la Garza, Vorster, and Parvin (1991, p.719), remains unrequited in its entirety despite far-reaching conversations, proposals, and hypotheses in the literature.

> [W]hether total float is perceived as a time contingency or as an incentive, its potential opportunity value is neutralized when consumed by owners. A revised model for pricing total float is needed. Such a model should allow for the trading of total float by making its commercial opportunity value explicit. The revised model should grant the contractor the right to administer total float, impose on the contractor the obligation to disclose its value and trade it on demand.

Similarly, Prateapusanond (2004) identifies total float allocation and management as two of the most important means of improving construction planning effectiveness, with the conclusion that the wrong course of action could lead to high implementation costs and the lack of achieving the project's goals. Therefore, "...a feasible and reliable total float allocation method should be of primary importance" (Prateapusanond 2004, p.16).

When evaluating the de la Garza et al. calculation for the trade-in value of total float, Pasiphol (1994) extends the precept that total float can be treated as “either incentive or monetary contingencies.” Given a fair opportunity to finish the project at the early completion stage (given an incentive to do so), or complete the project late (with appropriate penalty), Pasiphol contends that total float needs to be treated as a resource that can be traded. However, specific to the de la Garza valuation equation (Eq. 1.2), Pasiphol determines that:

> [T]his approach fails to consider that the total float belongs to all the activities of a path. In other words, the total float of the activity represents the possible maximum float time that an activity can consume. It is normally less that the maximum since other activities of the same path also are eligible to use it. *Consequently, the crucial problem is how to figure the exact amount of total float that an activity is entitled to so that the total float trade-in value equation can provide an accurate amount* [emphasis added] (Pasiphol 1994, p.77).

To overcome the lack of an appropriate theoretical framework to mitigate risk, analogies will be explored that address schedule elasticity and structure, schedule participants, and systematic schedule risk cost, each of which is rooted in unrelated research outside of the construction and engineering disciplines.

1.2.2 Unanswered Questions

De la Garza et al. (1991, p.727) aptly note that the development of mechanisms for trading total float rests with the research activities of academia:

> There is clearly a need to examine issues such as the trade-in value of float and many others relating to the practical implementation of CPM technology for managing construction …These issues must be addressed by academics who provide the [necessary] intellectual leadership…

What transcends previous research into float allocation and remains unfulfilled and recommended for further investigation is consistent across the participants in the dialogue and enumerated as follows (Prateapusanond 2004, Kim 2003, Pasiphol 1994):

1. The application of the research into practice by pilot test during an actual construction project.
2. Development of consistent mechanisms for the amount of float assigned to the parties (float allocation).
3. Creation of computer programs or subroutines for the introduction of float trading to scheduling programs.
4. Drafting of boilerplate contract language (clauses) for the implementation of float trading and allocation concepts

This research does not address these shortcomings in the body of knowledge or their application. Rather, it is recognizes that a much larger gap exists in prior research: *The lack of consideration for and use and/or inclusion of quantification and qualification theories that are rooted in rational, proven, and widely accepted research outside the bounds of the construction and engineering disciplines in the framework of risk analysis and mitigation.* Recognition of this overarching limitation is the stepping-off point for this research.

1.2.3 Research Questions

Since the de la Garza et al. seminal work, there has been intermittent research surrounding the pre-allocation of total float, and multiple attempts have been made to determine its value. While the concept remains appealing on paper and the appearance of solutions to the conundrum, the practicality of its application remains uncertain and an academic exercise. The body of knowledge lacks a comprehensive examination of the fundamental issues surrounding trading total float as a commodity. The absence of the measurement of where risk resides (risk being the precursor to float), the distribution of total float for consumption, and at what price, engenders the who, what, where, when, and how of *Risk Measurement, Allocation, and Pricing in Network Schedule Systems*.

In fulfillment of this objective, to present a comprehensive strategy for total float management, three objectives are established for which the following needs to be answered.

1.2.3.1 Research Objective 1: Location of Risk

Determine where risk is most likely to reside within construction project network schedule systems and how it can be measured. To achieve this, the following needs to be established and/or addressed:

1. What proven methodologies exist for determining where risk resides within an organizational structure?
2. What proven methodologies exist for determining where risk resides within or among a group of related unstructured entities operating in relation to each other?
3. Can these methods be adapted or translated to network schedule systems to bring meaning and/or clarity to risk identification and measurement for a meaningful purpose?

1.2.3.2 Research Objective 2: Allocation of Risk

Determine a method for allocating risk (or its manifest form within network schedule systems, float) among the participants and/or parties to a network schedule system. To achieve this, the following needs to be established and/or addressed:

1. What proven methodologies exist for allocating or apportioning something across an organizational structure?
2. What proven methodologies exist for allocating or apportioning something across a group of related unstructured entities operating in relation to each other?
3. What established processes are used and/or factors exist for determining/ reaching a decision regarding something within an organizational structure among a group of related unstructured entities?
4. Can these methods or processes be adapted or translated to network schedule systems to bring meaning and/or clarity to risk identification and measurement for a meaningful purpose?

1.2.3.3 Research Objective 3: Pricing of Risk

Determine a method for quantifying risk within network schedule systems by establishing a vehicle to set a price for the trading or exchange of float (specifically contract float). To achieve this, the following needs to be established and/or addressed:

1. What proven quantification/pricing methodologies exist to establish the price of risk within an organizational structure?
2. What proven quantification/pricing methodologies exist to establish the price of risk among a group of related unstructured entities operating in relation to each other?
3. Can these methods be adapted or translated to network schedule systems to accurately quantify/price risk in a meaning way?

1.3 Research Methodology

The methodology to support development of the research objectives and the resulting conceptual analogies is a multistep process rooted in the Scientific Method, which is traditionally accepted as iterations, recursions, interleavings, and/or orderings of (Jevons 1874):

- *Characterizations:* Observations, definitions, and measurements of the subject of inquiry.
- *Hypotheses:* Theoretical, hypothetical explanations of observations and measurements of the subject.
- *Predictions:* Reasoning including logical deduction from the hypothesis or theory.
- *Experiments:* Tests of the above described elements.

This is linearized as a pragmatic scheme in the following process: (1) define the question, (2) gather information and resources (observe), (3) form hypothesis, (4) perform experiment and collect data, (5) analyze data, (6) interpret data and draw conclusions that serve as a starting point for new hypothesis, (7) publish results for comment, and (8) retest (typically completed by others). In this research, the Scientific Method will be fulfilled in a four-part process, as described below.

1.3.1 Literature Review and Evaluation

A literature review is a description of the literature relevant to a particular field or topic. It is an account of what has been published on a topic by accredited scholars and researchers. It aims to appraise the critical points of current knowledge, including substantive findings as well as theoretical and methodological contributions to a particular topic. Literature reviews are secondary sources, are considered a piece of discursive prose, and as such do not report any new or original experimental work. Its ultimate goal is to bring the reader up to date with current literature on a topic and form the basis of understanding for another goal: continued, additional or future research that may be needed in the area being considered. According to Cooper (1988):

> [A] literature review uses as its database reports of primary or original scholarship, and does not report new primary scholarship itself. The primary reports used in the literature may be verbal, but in the vast majority of cases reports are written documents. The types of scholarship may be empirical, theoretical, critical/analytic, or methodological in nature. Second a literature review seeks to describe, summarise [sic], evaluate, clarify and/or integrate the content of primary reports.

This literature review is considered an exhaustive search and evaluation to achieve an in-depth level of understanding of the state of knowledge of: (1) risk quantification, via the Nobel Prize winning work of the Capital Asset Pricing Model (CAPM) of financial portfolio theory, (2) risk allocation via the research surrounding European voting allocations and the respective voting power of member states as found in the social decision-making realm, and (3) the price at which risk (or its most visible form, float) can be exchanged via the binomial valuation variation of real options associated with capital budgeting decisions.

1.3.2 Data Collection and Preparation

Data collection is the process of gathering and measuring information on variables of interest, in an established systematic fashion, that enables one to answer stated research questions, test hypotheses, and evaluate outcomes (Northern Illinois University 2004). Research data collection is common to all fields, including the physical and social sciences, humanities, business, etc. While methods may vary, and irrespective of the preference for defining data as either quantitative or qualitative, the emphasis on ensuring accurate and honest collection remains the same and is essential to maintaining the integrity of research.

Most et al. (2003) describe quality assurance and quality control as two approaches that can preserve data integrity and ensure the scientific validity of the results. Each approach is implemented at different points in the research timeline (Whitney et al. 1998):

- *Quality Assurance:* Activities that take place before data collection begins – its main focus is prevention (i.e., forestalling problems with data collection), where prevention is the most cost-effective activity to ensure the integrity of data collection.
- *Quality Control:* Activities that take place during and after data collection – identifies the required responses, or actions (detection and/or monitoring) necessary to correct faulty data collection practices and also minimize future occurrences.

Data collection for this research is limited to the creation and retrieval of simulated project and activity time (durations) from exemplar networks schedule systems to compare their as-planned versus as-built (simulated) values.

1.3.3 Mathematical Modeling and Analysis

According to Law (2007, p.6), process-centric modeling, commonly referred to as discrete event simulation:

> …concerns modeling the system as it evolves over time by a representation in the state variables change instantaneously at separate points in time. (In more mathematical terms, we might say that the system can change only at a *countable* number of points in time.) These points in time are the ones at which an event occurs, where an *event* is defined as an instantaneous occurrence that may change the state of the system. Although discrete-event simulation could conceptually be done by hand calculations, the amount of data that must be stored and manipulated for most real-world systems dictates that discrete-event simulations be done on a digital computer.

In this research, discrete simulation modeling will establish a mathematical model with the use of simulation, and in particular Monte Carlo Simulation, for: (1) allocating float (risk), (2) valuing float (risk) in its smallest increment (one-day), and (3) correlating the risk among a project's participants. These are jointly construed to consider the events, actions, and decisions that occur between identifying risks *a priori* and assessing project performance *a posteriori*.

1.3.4 Verification and Validation Process

Verification and validation is the process of checking that a product, service, or system meets its design specifications and that it fulfills its intended purpose. They are critical components of quality management and at their zenith are performed independently by a disinterested third party. According to the Capability Maturity Model (Paulk et al. 1993) and that found within the software quality control field, considered to have among the most robust verification and validation standards and processes (IEEE 1991), verification and validation are defied as follows:

- *Verification:* The process of evaluating to determine whether the products of a given development phase satisfy the conditions imposed at the start of that phase.
- *Validation:* The process of evaluating during or at the end of the development process to determine whether it satisfies specified requirements.

Validation ensures that the product actually meets the user's needs and that the specifications were correct in the first place, while verification ensures that the product has been built according to the requirements and design specifications. In other words, validation ensures that "you built the right thing;" verification ensures that "you built it right" (Lucko and Rojas 2010, Institute of Electrical and Electronics Engineers 1991).

In relation to this research, verification entails checking the mathematical model or models for accuracy and consistency via hand calculations and comparison to the computerized results, in order to: (1) determine the expected value of inputs and outputs, and (2) perform sensitivity tests by varying individual inputs and examining the behavior of the output for magnitude, patterns, and trend deviations; all to ensure the technical correctness of the concept and model. Validation encompasses comparing the results of the *a priori* calculations generated by the model with actual project time/cost performance (as-built schedule updates and other project records) *a posteriori*.

1.3.5 Simulation

1.3.5.1 Monte Carlo Methods

Monte Carlo methods, Monte Carlo Simulation (MCS), or simply ‘simulation’ is probably the most common predictive technique used to model possible outcomes in science, engineering, management and finance. Developed by von Neumann, Ulam and Metropolis in the 1940s Los Alamos National Laboratory weapons research program, the method was named for the famous European casino (Metropolis 1987). MCS methods are a class of computational algorithms that rely on repeated random sampling to obtain their results and are most suited to calculation by computer. Simulations are used when it is infeasible or impractical to compute an exact result of a complex system with a deterministic algorithm.

More recent studies have applied MCS to overcome the limitations of deterministic methods (Elkjaer 2000, Lee 2005, Ranasinghe and Russell 1992). MCS is designed to represent uncertainty (risk and/or variation), associated with some of the parameters in the system under study with the use of random numbers derived from predetermined estimated probability distributions.

The principal application of MCS is to study the behavior of stochastic processes – problems in which the input is of a random nature. MCS is particularly effective when the process is nonlinear or involves many uncertain inputs. These inputs may be distributed differently from each other (Hartford and Baecher 2004). When simulations have been used to model space and oil exploration, their predictions of failures, costs, and schedule overruns are routinely better than human intuition or alternative heuristic (‘soft’) methods (Hubbard 2009).

The importance of risk assessment in decision making using Monte Carlo simulation was presented early on in the *Harvard Business Review* article, “Risk Analysis in Capital Investment” (Hertz 1964). Van Slyke (1963) addressed typical problems in the program evaluation and review technique (PERT) with simulation. Because of the relative complexity in calculating total project duration from probabilistic estimates of component work packages, Monte Carlo simulation based on network schedules has been intensively investigated (Finley and Fisher 1994, Lu and AbouRizk 2000, Hulett 1996, and Lee 2005).

1.3.5.2 Monte Carlo Methods in Practice

MCS generates a large number of sets (iterations) of randomly generated values for the uncertain parameters (variables) and numerically computes and records the performance of each set. In this research, each MCS iteration will produce a deterministic CPM schedule. After several hundred, thousand, or in some instances million iterations, the MCS will produce a data summary that provides information about its range of variability.

From this randomly generated set of event statistics, an itemization of the cumulative distribution function (CDF) (the frequency of occurrence of a random variable) is produced, and estimated statistics such as the expected value, variance, and correlation are generated. Irrespective of the number of stochastic inputs, each MCS iteration or ‘run’ results in a single observation of the process or problem. Increasing the number of stochastic input variables does not increase the number of iterations necessary for a desired level of accuracy. Rather, that can be established by applying methods of statistical inference.

The number of iterations required in a simulation varies depending on the size and complexity of the model being considered. One method to determine an adequate number of iterations surrounds keeping track of the stability of output being generated. Heuristically, as additional iterations are included in a simulation, the output distributions become more stable. This stability occurs because the statistics that describe the distributions being evaluated vary less as additional samples (data from individual runs) are obtained.

1.3.5.3 Network Schedule Systems and Monte Carlo Simulation

As mentioned above, MCS offers a viable alternative when analytical models are mathematically intractable or must be oversimplified. CPM scheduling, that is multi-process scheduling, precedence constrained scheduling, scheduling with individual deadlines, scheduling with probabilistic and/or conditional branching, etc., fits this categorization (Kim 2007).

Barraza et al. (2004) conducted a study of probabilistic forecasting of project duration and cost using network-based simulation. In the study, the correlation between past and future performance is simplified by adjusting the parameters of probability distributions of future activities with the performance indices of finished works. Lee (2004) also presented a network-based simulation approach to compute the probability to complete a project in a specified time. In the aligned cost estimating area, Monte Carlo methods have been tested as a method for dealing with correlation between random variables (Chau 1995, Touran 1993, Touran and Wiser 1992).

1.3.5.4 Modeling Distribution and Uncertainty

When known performance data are available, probability density distributions or functions (PDFs) can be approximated using general techniques such as the ‘method of moments’ (the statistical estimation of population parameters such as mean, variance, median, etc.) by equating sample moments with unobservable population moments and then solving those equations for the quantities to be estimated, ‘maximum likelihood estimators’ (MLE) (an estimate based on know parameters that when applied produces a distribution that gives the observed data the greatest probability of occurrence), and the ‘ordinary least squares’ (OLS) method (based on the unknown parameters in a linear regression model and the sum of squared vertical distances between the observed responses and the responses predicted by the linear approximation).

When data is not available, subjective estimates of PDFs can be considered for risk analysis models. It is important to acknowledge the importance of subjectivity; even if objective data form the basis of a forecast, judgments are exercised in the various adjustments that are made to produce the estimate for the project being modeled (Fellows 1996).

1.3.5.5 Construction Industry Modeling

In the construction industry, several PDFs are considered adequate for modeling activity durations and construction operations (Arízaga 2007). Predetermined probability distributions for activity durations include beta, triangular, normal, and uniform distributions; for cost, lognormal, triangular, pearson-type and beta distributions are preferred.

Several studies compare the use of different distributions in risk assessment models. However, the results present mixed opinions. Fente et al. (2000) claim that most of the construction data best fit the beta region and therefore present a methodology for the estimation of the beta parameters. Conversely, Wilson et al. (1982) studied the use of beta versus triangular distributions on ground operations, concluding that there were not significant differences in the simulation outputs. Schexnayder et al. (2005) provide more details on the determination of beta parameters for construction operations.

Another example presented in Touran (1997), where results of sensitivity studies of the use of normal and lognormal PDFs on tunneling operations did not show any statistically significant difference in the predicted mean completion time. Touran (1997)studied the effects on simulation results of different PDFs for construction simulation models, where beta PDFs were used to define the probabilistic duration of construction activities.

Back et al. (2000) studied the determination of triangular distributions from historical cost data. They claim that beta and triangular distributions are the most suitable. However, due to the more complicated process of the calculation of the beta parameters and its variety of shapes, the triangular distribution is preferred (Arízaga 2007).

1.4 Scope Limitations and Assumptions

Several limitations exist within precedent work that remains appropriate to the research. Similarly, in fulfillment of this research, other limitations and/or exclusions are made for clarity and to address similar elements in the body of knowledge that may traverse different paths, identify other theories, and convey conclusions serving different purposes than herein.

1.4.1 Total Float

Float is characterized by multiple definitions depending upon its location within a schedule system, e.g. free float, interfering float, back float, safety float, etc. Given that total float is assigned to no specific entity and that other categorizations or characterizations of float may be limited in their application to an individual schedule activity or a few activities along a chain of events or occurring on parallel paths, and the purpose of this research being in part the development of a *framework for the fair quantification, allocation, and valuation of float to mitigate risk in projects*, only float with widespread availability can meet be engaged to this supposition. Total float as it is shared by all activities along a path meets fulfills this obligation. Moreover, post CPM duration float aggregated by the project owner or held by the scheduler beyond the confines of the network schedule (in construction the general contractor) and of the variety later defined by this research as 'contract float,' is the singular float type germane to this research. Hence, all other float variations are excluded from consideration.

1.4.2 Critical vs. Non-Critical Activities

Total float by definition can reside in multiple locations within a network schedule system, except along the critical path. Also, critical activities are by definition afforded no float of any description. Previous research singularly focused on allocation methods and the potential to exchange float among those participants of a network schedule with the ability to control and/or expend float, namely non-critical activities. This research seeks validation of a vehicle to reduce risk introducing allocated total float (contract float) for the specific use of critical activities. Hence, this research excludes extension of the float allocation concepts, practices, and methodologies resulting from research associated with non-critical activities. It is entirely focused on research involving float found beyond the network schedule system and allocation it along the critical path for control by critical activities.

1.4.3 Zero or Negative Float

The allocation of float represents a vehicle for the reduction of risk across schedule participants. The allocation or pre-allocation as it may be characterized of total float (specifically contract float) is an activity undertaken during initiation of a project and an outflow of schedule development, at which time total float is positive or zero. Zero float precludes allocation altogether. Similarly, it is left to individual judgment based upon the size, complexity and duration of a network schedule to determine what is an appropriate threshold for the minimum contract float at which to enter the allocation/pre-allocation process.

Negative float (defined as occurring when activities have consumed more time than allotted and the project is deemed to be behind schedule, having consumed all available float), is anathema to float allocation for risk mitigation purposes. To encounter negative float, a project must be active and beyond the point of allocation or it cannot be demonstrated to be capable of completion within its allotted duration. Either case is outside the bounds of consideration herein.

1.4.4 Preceding Methods

Antecedent research demonstrates that total float can be pre-allocated by contractual clause beyond that traditionally included in today's construction contracts. Several model clauses direct the use of total float and explain the manner in which responsibility for delays should be apportioned. They include: (1) The Float Definition clause, (2) the Pre-Allocation of Float clause, (3) the No Damage for Non-critical Delay clause, and (4) the Formulas clause. This research recognizes that contractual provisions are necessary to exact the allocation of total float among the parties, but does not venture into the legal arena in furtherance of contractual language specific to the conclusions and methods defined herein. Rather, this research is an alternative to the model provisions and equal allocations by fiat previously proposed and will leave alone the contractual component(s).

1.4.5 Critical Chain Project Management Method

The precept of this research is that the allocation of total float to critical activities is a method for risk reduction in network schedule systems. CCPM maintains a contrary position, in that it seeks the accumulation of float from activities based upon a probabilistic expectation and level of confidence for the duration an activity will require. CCPM then aggregates all surplus float from the 50% confidence level duration into a single buffer at the end of the project. CCPM is the inverse of that hypothesized herein and requires the introduction of resource monitoring into the process. Accordingly, this research will not build upon the tenets of CCPM. However, CCPM accomplishes its aggregation of a singular post-schedule float buffer through the use of mathematical modeling, a concept warranting investigation, introduction and possible inclusion herein.

1.4.6 Lean Construction Principles

Generally, Lean is a production practice that considers the expenditure of resources for anything other than the creation of value for the end customer or user to be wasteful and is therefore a target for elimination. It is centered on preserving value with less work. When applied to the construction industry, Lean represents an holistic project management system, not a scheduling technique or premise, nor is it focused on risk mitigation. Rather, Lean Construction remains true to its roots by seeking value generation and fits within the bounds of the four strategic elements or notions of Lean (Pettersen 2009):

1. Lean as a fixed state or goal (being lean).
2. Lean as a continuous change process (becoming lean).
3. Lean as a set of tools or methods (doing lean/toolbox lean).
4. Lean as a philosophy (thinking lean).

Hence, this research is viewed as an element of item 3 above – part of the set of Lean tools to be engaged in fulfillment of the Lean (construction) philosophy.

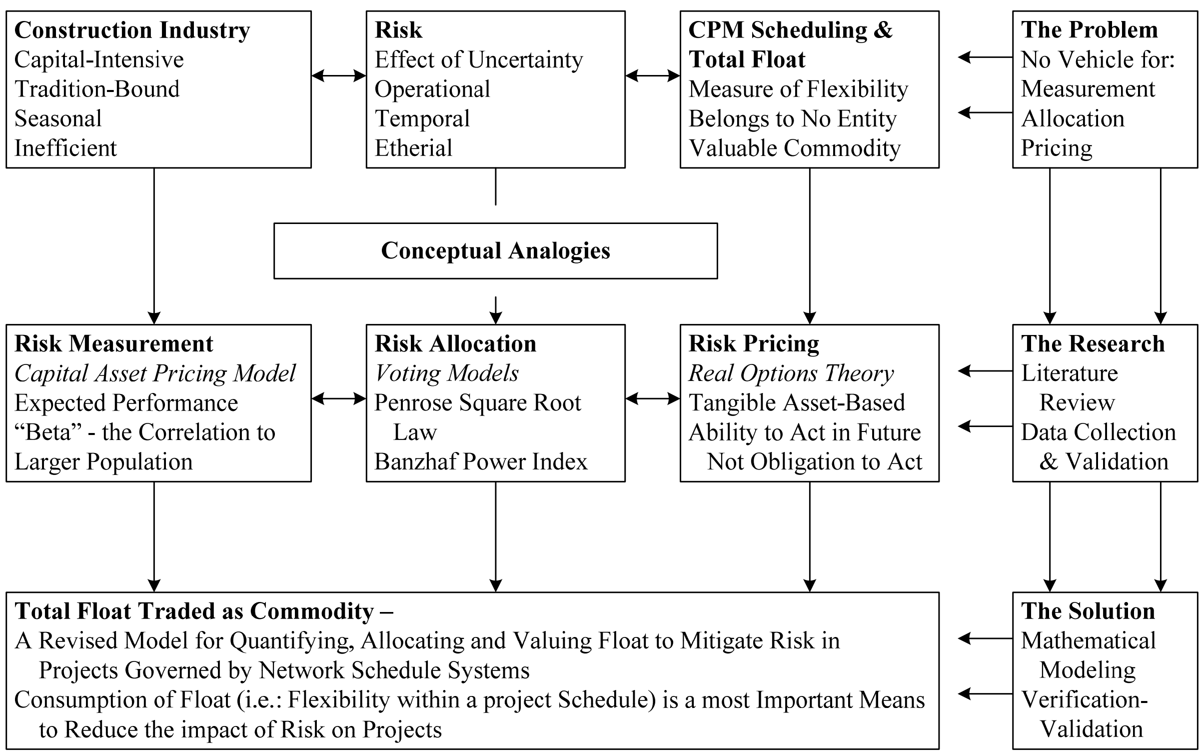
1.5 Definitive Scope Statement

The purpose of this research is to introduce a comprehensive theory for the management of total float within network schedule systems. In particular, its goal is to develop methods, based on proved analogous research theories and practices, to measure (quantify) risk, allocate risk among the network's participants, and price risk for exchange between participants and/or activities operating within network schedule systems. Based upon this scope, the primary limitation of this research in fulfillment of de la Garza, Vorster and Parvin's (1991) notion for "Total Float Traded as Commodity" is that it singularly and ubiquitously applies to total float. Ultimately, the aspiration of this research is to contribute to the current body of knowledge in the scheduling and construction arenas to improve overall project performance through the mitigation of risk and uncertainty.

1.6 Dissertation Organization

This dissertation was produced in accordance with guidelines that permit the text of original papers submitted for publication as the main part of the dissertation. It consists of a total of five chapters incorporating three separate but mutually dependent papers, introductory information, and concluding matter. Each chapter is self-contained with all relevant sections – abstract, introduction, literature review, methodology, conclusions and recommendations, references, etc. These chapters are divided as follows:

Figure 1.2: Dissertation Process Diagram



1.6.1 Chapter 1 – Introduction

This chapter presents the motivation for the research, background information and basic understanding of the concepts spanning the three articles (Chapters 2, 3, and 4), the research objectives and methodologies, and the antecedent work from which this research emanates.

1.6.2 Chapter 2 – Financial Portfolio Theory as a Measure of Risk Correlation of the Participants in Network Project Schedules

This chapter, the first article in the series, centers on the inherent risk presented by those operating within the network, known as *schedule risk* (the likelihood of failing to meet schedule plans and the effect of such failure). It examines the use of *Beta* (β) (the risk correlation of an individual stock to that of the entire market), from the Capital Asset Pricing Model (CAPM) of financial portfolio theory to determine parallels with respect to the inner workings and risks represented by each entity or individual activity within a network project schedule system as an appropriate measure of their risk to the overall system.

1.6.3 Chapter 3 – Measures of Decision-Making Power for Quantifying Risk Allocation within Network Project Schedules

This chapter, the second article in the series, centers on risk as it is viewed through network flexibility, known as *schedule float* (the aggregate time an activity may be extended or delayed without impeding overall outcome). It is explored using analogous voting allocation and voting power research from the Penrose square root law and the Banzhaf power index.

1.6.4 Chapter 4 – Real Options as a Model for the Monetization and Consumption of Flexibility within Network Project Schedules

This chapter, the third article in the series, centers on the consumption of float using the binomial valuation variation of *real options* (defined as non-financial options,(not derivative-based traded instruments, surrounding tangible assets that creates a future right of choice, but not an obligation, to pursue a decision). Real options theory will be developed as the monetization component for trading float.

1.6.5 Chapter 5 – Summary and Conclusions

This chapter connects the main points and seminal concepts presented by this research individually presented in the three articles. It connects their individual contributions to the overarching precept for a working model to measure, allocate, and price risk as the basis for trading total float as a commodity.

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CHAPTER 2

Financial Portfolio Theory as a Measure of Risk Correlation of the Participants in Network Project Schedules

Abstract. Risk is ever-present in all construction projects where it resides among the collection of subcontractors and their array of individual activities. It manifests itself in different ways in projects of all sizes and complexities. Wherever risk resides, the interactions of participants becomes paramount in the way that risk is viewed and addressed. Within a network project schedule, inherent risk becomes recognizable, quantifiable, and can be mitigated by consuming float – the flexibility of a project to absorb delays. Allocating, owning, valuing, and expending float has been discussed since the inception of the critical path method. This research investigates the initial element of a three-part treatise that examines how float can be traded as a commodity, an unrequited concept in construction engineering and management whose promise remains unfulfilled for lack of a holistic approach. The Capital Asset Pricing Model (CAPM) of financial portfolio theory describes the relationship between systematic risk and the expected return of individual stocks. It is explored as an analogy to the intrinsic risk of the participants in construction projects. Their relationship and impact on schedule performance, schedule risk (the likelihood and the effect of failing to meet as-planned schedule duration), are matched with CAPM's beta component, the correlation of an individual stock to that of the market, to determine parallels for risks represented by each schedule activity. This research represents the theoretical extension required for the identification and correlation of risk to develop a trading model across a network schedule system's critical participants.

2.1 Introduction

In his magnum opus *The Theory of Moral Sentiments and An Inquiry into the Nature and Causes of the Wealth of Nations* ("The Wealth of Nations") (Smith 2003a), considered to be the first modern work of economics, Scottish social philosopher and father of modern capitalist economics Adam Smith (1723 – 1790) defined a perfectly competitive market as "one in which there is no impediment to free contracting and free entry and exit of productive resources" (Smith 2003a, p.126). He theorized that a perfect market is one in which companies can enter and leave as they wish, because no single firm holds power over the market; as one leaves, another will either enter or a different one will form.

In theory, network schedule systems correspond to this definition and perform in a perfectly competitive manner: No single activity, no single participant may be large enough to set the schedule pace of a homogeneous project being completed. However, because the conditions for perfect competition to exist are strict, like perfectly executed schedules, perfectly competitive markets and schedule systems are scant. The interaction of buyers and sellers in the marketplace and activity owners within network schedule systems correlate to one another against the benchmark of perfection.

It is this interaction, the correlation of market participants one to another is of interest as an allegory to the inner workings of network schedule systems and the correlation of their actions and ultimate schedule performance.

2.2 Literature Review

2.2.1 Financial Markets

The mechanism that fosters ability to participate in the ownership and exchange of assets is known in economic terms as the financial markets or collectively as ‘the market.’ In broad terms, the financial market is any marketplace where buyers and sellers can participate in the trade of assets. It has become abstract in its meaning and ubiquitous in its application. Formally, the market is defined as the aggregate of possible buyers and sellers of a certain good or service and the transactions between them (WebFinance 2011f) and is characterized by transparent pricing and basic regulations. It allows people and entities to bilaterally trade, i.e. to buy and/or sell financial securities such as stocks (the element that signifies ownership in a corporation and represents a claim on part of the corporation’s assets and earnings (WebFinance 2011k)), or bonds (debt investments by which an investor loans money to an entity (a corporation or government) for a specific time at a specified interest rate (WebFinance 2011b)), commodities (a raw material capable of being bought or sold such as agricultural goods, precious metals), and other fungible items (WebFinance 2011d) like oil and gas (bulk goods or commodities whose individual units are capable of mutual substitution), i.e. an item where it is of no consequence whether the same item is returned when completing a transaction. For example, a barrel of North Sea crude oil is the same regardless of whether it was the individual barrel and/or barrels that initiated the transaction.

Within financial circles, the market facilitates raising capital (capital being defined economically as the already-produced durable goods used in production of goods or services exclusive of land; by definition, capital must be produced by human hands before it can become a factor of further production (WebFinance 2011c)), the transfer of risk (the potential that a chosen action (including inaction) will lead to a an undesirable outcome within the market resulting in a financial loss (WebFinance 2011j), the transfer of liquidity (the speed and ability for an asset to be sold without causing a significant change in the price and/or with a minimum loss of value (WebFinance 2011g)), i.e. how quickly and cheaply an asset can be converted into cash and the trading and/or conversion of currency (e.g. changing Japanese yen (JPY, ¥) into United States dollars (USD, $) by a market trade based upon the current exchange rate). Table 2.1 indentifies the generic types of markets and their purposes.

Table 2.1: Types of Financial Markets

Market	**Description / Function / Purpose**
Capital Markets	Primary Markets for the initial issuance (selling and buying) of stock, and Secondary Markets for buying and selling of previously issued shares of stock
Stock Market	Provides financing for corporate entities through the issuance of shares of stock and enables the subsequent trading of shares
Bond Market	Provides financing for corporate and governmental entities through the issuance of bonds and enables the subsequent trading
Commodity Market	Provides the vehicle for the purchase or sale of commodities
Money Market	Provides short-term financing by direct investment (debt)
Derivative Market	Provides the vehicle to trade financial instruments to manage and/or transfer risk
Futures Market	Provides a standardized vehicle for trading commodities and other financial assets at a future date
Foreign Exchange Market	Provides the vehicle for the changing of one currency into another
Insurance Market	Provides the vehicle to distribute risk across multiple entities

2.2.2 Functional Markets

The market or 'stock market' hereinafter described in general is different from its functional components known as stock exchanges. While the stock market refers to the totality of opportunities to bring together buyers and sellers, a stock exchange is a corporation or mutual organization that is in the business of uniting individuals and/or organizations to exact transactions (buy and sell trades) for individual stocks or other financial instruments. Each market identified within Table 2.1 is supported by at least one exchange. From a functional perspective, the stock market is made up of multiple stock exchanges characterized as "[a] defined market where specialized intermediaries buy and sell securities under a common set of rules and regulations through a closed system dedicated to that purpose" (Michie 1999, p.3).

The purpose of modern financial markets is twofold (Petram 2011): price discovery (the innate ongoing process by which the market seeks to determine the price at which a asset will trade) and provision of liquidity (the state whereby assets can be bought and sold at will without causing large price movements in the market). From the opposite perspective, the market for a particular asset is considered liquid if trading the asset does not cause significant changes in price. The interaction of traders in the marketplace determines the price of the assets that are traded on the exchange specific to the asset. The price discovery and liquidity functions are a result of the concentration of traders operating in the specific exchange and across the broader financial market.

2.2.3 History of the Stock Market

In 2010 the earliest known stock certificate was discovered by historian and scripophile Schalk (Stock Market Story 2011). He uncovered a 1606 document made out to "Enkhuizen inhabitant 'Pieter Hermanszoon boode' ...real name Pieter Harmensz who served as a personal assistant to the Enkhuizen mayors" (World's Oldest Share 2010, p.1). It was issued by the VOC chamber of Enkhuizen, an initial subscriber to the offering for investment in the *Vereenigde Oost-Indische Compagnie* (the Dutch East India Company or the United East India Company) commonly referred to as the VOC, the 17th and 18th centuries' largest trading company.

Schald found the document (originally housed in the in the Enkhuizen city archives but discovered in Germany) while researching his Utrecht University thesis. Dated September 9,1606 it is three weeks older that the previously recognized oldest share and recorded the final installment of a 150 guilder (*ƒ*) investment in VOC, the world's first joint-stock limited liability company with freely transferable shares (World's Oldest Share 2010). Public share subscription (the initial public stock offering limited in its breadth) in VOC ended on September 1, 1602, thereby allowing everybody the chance to participate in the new venture by securing an ownership interest in a secondary market.

Among the resulting 538 Enkhuizen subscribers (investors) were craftsmen, entrepreneurs, and citizens like Harmensz. The VOC chamber of Enkhuizen invested 540,000 guilders and was the third largest subscriber to the offering, following Amsterdam and Middelburg. From this investment, Harmensz purchased his 150 guilder share, reportedly in installments. This gave him the right to receive dividend payments up to 1650 AD.

2.2.3.1 Early Markets

The larger financial markets took centuries to develop. The first recognized concept dates back to the earliest recorded history in the ancient world: clay tablets recording the interest-bearing loans of ancient Mesopotamia (Silber 2009). Exchangeable bonds did not materialize until much later during the Italian medieval and Renaissance periods. Much of the earliest business of these financial markets dealt with stocks and securities (bonds) involved with agriculture, shipping, and in particular the spice trade. The earliest formal financial markets trace their origins to the 12th and 13th centuries, with some scholars crediting ancient Rome as the earliest 'share market' trading venue (Malmendier 2005):

The Roman Empire (1st century): The ancient Romans developed a sophisticated financial system surrounding government leaseholders, known individually as *publicani* or collectively in their corporate form as *societies publicanorum* (in loose terms a contractual union of individuals formed to promote a common purpose (Malmendier 2005, 2009)). It relied on the use of intermediaries, *argentarii* (bankers) and *proxenetae* (brokers), who pooled and distributed funding and resources across the Roman economy (Temin 2004). Not only did the existence of *proxenetae* connote the existence of ownership stakes or 'shares' in the *societies publicanorum*, but the Roman philosopher and statesman Marcus Tullius Cicero (Cicero 106 BC – 43 BC) mentions *partes* or shares, *participes* or shareholders, and *societatum publicanorum* the term for individual citizens possessing shares multiple times in his speeches (Malmendier 2005).

Malmendier (2005) posits that Cicero implied the existence of a market for the trading of *partes*, when referencing *magnae partes* or large shareholders, thereby identifying disparate ownership between parties. Similarly, historian and author Valerius Maximus, who wrote during the reign of Roman Emperor Tiberius (14 AD to 37 AD) mentions the *particula* or the little share of T. Aufidius (Maximus 2004), further supporting the implication of shares of different nominal values.

The existence of varying investment interests addresses one component of a functioning financial market or stock exchange. The second is the transfer of shares at different prices depending upon the performance of the *societies publicanorum*. Again Cicero is credited with providing such insight. Within his second speech against Verres, Cicero quoted the exceptional restriction, *"'Qui de L. Marcio M. Perperna censoribus redemerit ...socium non admittito neve partem dato neve redimito.'* [As translated] that is, anyone who had been leasing under the censors L. Marcius and M. Perperna was not admitted to the current lease, neither as a partner, nor as a shareholder, nor should he be allowed to buy any shares later" (Malmendier 2005, p.38). This revealed that shares were traded among multiple parties. He continued in other speeches to cite the varying price structure of shares when speaking of *"partes illo tempore carissimae,"* or shares that had a very high price at that time (Malmendier 2005, p.38), evidence of stock price fluctuation and to the existence of a 'stock-market life' in ancient Rome.

Beyond this, Braudel writes in *The Wheels of Commerce* that, "all the evidence points to the Mediterranean as the cradle for the stock market" (Braudel 1982, p.101), as most Italian cities farmed out the collection of taxes to the *monti* (organizations that bore close similarity to the Roman *publican*). The *monti* dealt with capital divided into shares called *luoghi*. By the 15th century *luoghi* transactions occurred alongside that of the bond or *prestiti* hereinafter described (Smith 2003b). This arguably became the first fully fledged stock market.

Venice (12th to 14th century): As early as 1171, the Republic of Venice began borrowing (via conscription) from its citizenry as it became concerned about its war depleted treasury. This debt, known as *prestiti* for "to lend" carried a 5% interest rate on an infinite maturity date (Silber 2009). While originally met with contempt and suspicion, the government debt ultimately became a valuable investment, and a market soon developed for its exchange, the first bond market.

This continued as a fixture within Venetian society, and the credibility of debt instrument became less suspect and solidified as an investment. Between 1262 and 1379, the Republic of Venice never missed an interest payment (Silber 2009). Based upon Venetian success, Pisa, Verona, Florence and Genoa also entered the debt market with the issuance of their own 'war bonds,' as well as initiating trading in shares of individual companies. The *Famiglia de' Medici* or House of Medici, textile traders with their origin in the 14th century Republic of Florence, formed the Medici Bank, the first to engage in widespread trading of bonds across Italy, eventually becoming 15th century Europe's largest and wealthiest family institution (Padgett and Ansell 1993); producing four popes of the Catholic Church over a century – Pope Leo X (1513 – 1521), Pope Clement VII (1523 – 1534), Pope Pius IV (1559 – 1565), and Pope Leo XI (1605).

Prestiti interest payments were suspended in the 1380s due to war between Venice and Genoa. When the bond market returned post-war, the interest rate paid was reduced, causing a significant drop in the price at which Venice's bonds were traded. Further turmoil and challenges to the stability of an expanded European bond market resulted from the Black Death pandemic (1348 – 1350) and the Hundred Years War (1337 to 1453), the series of separate wars whose major events were: The Edwardian War (1337 – 1360), the Caroline War (1369 – 1389), the Lancastrian War (1415 – 1429), and the slow decline of Plantagenet fortunes after the appearance of Joan of Arc (1412–1431) (Allmand 1998) waged between the Kingdoms of England and France. This caused monarchies to default on their debts to the Italian banks.

However, the concept of sovereign debt as a tradable commodity endured as the forefront of financial innovation shifted from its origins in Italy to Europe. Its stability is highlighted by the 1351 prohibition under Venetian law of spreading rumors with the intention of negatively impacting the price at which sovereign debt traded, one of the earliest laws aimed at potential market manipulation (Cessay 2006).

France and Belgium (12th and 13th centuries): In 12th century France, the *courratiers de change* or the money lenders and money changers (typically found across history as those completing the transactions associated with the temples and/or churches that functioned as the banks of their time) were focused on the handling and regulation of the agricultural debt on behalf of the banks. Their familiarity with the role of intermediary and with financial trading and exchange lends credibility to their being ascribed as the first brokers.

By the end of the 13th century, formal trading began in Belgium. Traders held informal meetings in an Antwerp building owned by a man named Van der Beurse (though it is often mistakenly believed that this trading occurred in a Van der Beurse house in Bruges). It is in Belgium that we find the origins of the "bourse," that has become internationally ubiquitous and synonymous with the expression 'stock market.' This bourse emanated from the counting houses operated by the Hanseatic League (an economic alliance of mercantile cities founded by the Germans and Scandinavians that extended from the Baltic to the North Sea) in order to expedite trade in Bruges and Antwerp.

By 1309, this group of traders became known as the *Brugse Beurse*, either from a sign outside a trading center showing one or a few purses (*bursa* is Latin for bag), or perhaps because the merchants / traders gathered at the Van der Burse facility, though nobody is certain. Similarly, financial trading spread to adjacent counties like Flanders, Amsterdam and Ghent where the trading branches became known as *Beurzen* (Silber 2009, Economy Watch 2010). The 17th century writer Samuel Ricard, author of *The New Businessman*, is credited with specifically defining the term bourse to mean "exchange" or "stock exchange" with this expanded definition: "the meeting-place of bankers, merchants and businessmen, exchange currency dealers and bankers' agents, brokers and other persons" (Braudel 1982, p.97).

Europe (16th century): By the 16th century mechanisms for trading financial assets existed in several locations across Europe and the Italian peninsula. Financial assets like stocks and bonds existed in different forms, but the bulk of financial transactions were carried out in precious metals (Smith 2003b). During this time, the concept of a 'trade fair' (a timed event at a central gathering location with special exemptions from the heavy duties and taxes typically imposed on commercial transactions) came into being. Transactions at these fairs were best described by Braudel (1982) as taking the form of a pyramid:

> Markets in many local goods, usually cheap and often perishable, formed the broad bottom of the pyramid. Higher up in the pyramid were smaller numbers of more expensive luxury goods, often transported from far away. And finally, at the very top, was an active money market controlled by a few major dealers." (Smith 2003b, p.13).

For example, at the Leipzig fairs, shares of German mines changed hands, while near Paris municipal bonds IOUs and lottery tickets were traded (the lottery ticket originated in the Middle Ages as a form of government bond that included the possibility or 'chance' to increase the interest rate and win a large prize). In Antwerp, two fairs were formally scheduled each year in the spring and fall. However, its year-long tax exemption was perceived as a continuous fair (Ehrenberg 1963, p.309). To take advantage of the favorable commercial conditions, many 15th and 16th century merchants established permanent presences in these cities, thereby concentrating early financial markets in select cities across Europe and the Mediterranean.

As conflicts (wars) moved from being between neighboring counties, cities, towns, etc. to being international (across oceans and/or continents), merchants were forced to commit their funds for the necessities of their business for longer durations of time at ever-increasing sums, and with more participants. Bills of Exchange, basic IOUs negotiated between merchants, became necessary to fund and protect the claims of others on individual business inventories, infrastructure (ships, tools, machines, etc.) and eventually became a *de facto* currency for which an independent market for exchange developed. Also created during this period out of necessity, as rulers sought to borrow large sums of money for longer and longer periods of time to finance their wars, was the annuity (an item whose popularity lasts to this day). Charles V, Holy Roman Emperor, House of Hapsburg is credited with conceiving of the annuity whereby the government would agree to make payments at a fixed interest rate to the purchaser (the one loaning money to the government) for the life of the purchaser (Smith 2003b). Some variations were capable of passing from generation to generation. As both types of annuity were transferable between purchasers, a market for annuity contracts developed.

The Dutch (17th century): In 1602, the Dutch East India Company, the VOC, was formed when the Netherlands granted it a 21-year monopoly to carry out colonial activities in Asia. In 1623, the States-General granted a permanent charter to VOC; it would stay in business with its stock remaining active in the secondary market for over two centuries. It was history's second multinational corporation after the British East India Company that was formed two years earlier (the British had begun experimenting with joint-stock companies by the late 1500s, e.g. the Muscovy Company) and sought to wrestle trade with Russia away from Hanseatic control (Petram 2011).

Joint-stock companies differed from the traditional partnership form of ownership typically used by merchants, and the VOC was the first company to issue shares of stock that were easily tradable. Adam Smith is credited with noting that joint-stock companies were formed to overcome the problems of distant trade that consumed and/or tied-up large quantities of capital for long periods of time. To function as an international merchant generally required more capital that any single merchant or merchant partnership could afford. Joint-stock ownership or funding transferred that risk to a broad spectrum of invertors not partners, "making the firm itself an entity independent of its owners," (Smith 2003b, p.16), that were participants in company profits, but not decision-making.

"'This little game could bring in more money than contracting charter parties for ships bound for England,' wrote Rodrigo Dias Henriques to Manuel Levy Duarte on 1 November 1691" (Petram 2011, p.1). The game to which Henriques was referring was the trading of VOC shares and derivatives (financial dealings contracts that take a share in something, e.g. stocks, commodities, currency, etc. as its financial valuation basis, but that do not necessarily trade the actual share; it is a financial transaction 'bet' of sorts with predetermined variable and specifics between two entities (WebFinance 2011e)).

Henriques acted as an exchange agent for Duarte on the Amsterdam market and regularly performed a high number of transactions on his behalf. He speculated on share price movement and managed Duarte's portfolio to reduce risk. This secondary market for VOC shares became the first modern securities market in the world. It was in 17th century Amsterdam that the global securities market took on its current form (O'Hara 2003).

The VOC capital subscription was a great success; in Amsterdam alone, 1143 investors signed up for ƒ3,679,915.60 (guilders, about €100 million or between $130 – $150 million). According to a clause on the first page of VOC's subscription book, shareholders could transfer their shares to a third party at any time (Petram 2011). The modern stock market had begun. By 1610, due to increased trading in VOC, a new exchange was opened. It traded commodities, currency, shareholdings (stocks), maritime insurance, called actions (futures contracts), and margin loans (Braudel 1982).

The British (17th century): By the 1690s, a robust market for joint-stock shares came into being in England (Smith 2003b). It was a result of the ascension of Dutch ruler William of Orange (1650 – 1702) to the throne of England, Scotland, and Ireland after the Revolution of 1688, also known as the "Glorious Revolution." Dutch stadtholder William III of Orange-Nassau lead the Dutch fleet and army in support of English parliamentarians and overthrew King James II of England (1633 – 1701). He used concepts from his homeland to modernize England's finances. Soon thereafter, the Bank of England issued the kingdom's first government bonds to pay for the war. This lead to a flurry of subscriptions as English joint-stock companies began issuing shares and a secondary market began. It did not use the Royal Exchange established in 1571, because stock brokers were not permitted in this exchange due to their rude manners.

More than stock was traded on the English market. Brokers traded government bonds and annuities (also called government stock). "Transactions took place in coffeehouses clustered among the twisted warren of narrow streets called Exchange Alley" (Smith 2003b, p.20). With heightened interest in the market and for trading of shares on the British East India Company, the newly created Bank of England and other companies like the Hudson's Bay Company resulted in an ad hoc financial press of sorts. It was an English broker named John Castaing who operated out of one of the coffeehouses who began posting regular listings of the prices at which stocks and commodities traded; the beginnings of the London Stock Exchange (Michie 1999).

2.2.3.2 Stock Markets in the United States

The history of the stock market in the United States dates back over two centuries. Much like the antecedent European and Mediterranean markets, the origins of the financial markets in US began with the need to finance a war. The fledgling colonial government sold $80 million in post-Revolutionary War refinancing bonds (Geisst 2004), promising to return a future profit. Coincident to this, privately-held banks in the US began issuing shares to raise capital; the US stock market had begun. In the 1790s, a group of large merchants entered an agreement and created a formal stock market: the New York Stock Exchange (NYSE).

The agreement that began the NYSE, the Buttonwood Agreement (named for the Buttonwood tree under which the twenty-four signatories met) signed on May 17, 1792 outside of 68 Wall Street, set the stage for the organized trading of five securities in New York City. The first listed company on the NYSE was the Bank of New York (Terrell 2010). Under the agreement, the merchants committed to meet daily on Wall Street to trade stocks and bonds.

From the mid-1880s through the Industrial Revolution, due to rapid economic growth, companies needed a source of capital to expand and meet the increased demands of a modern society. The US stock market facilitated corporate and societal expansion, and the potential of stock transactions became valuable to both investors and companies (Geisst 2004). Not only did the Industrial Revolution change the face of society, it changed the market as well. A new form of investing began when it was realized that profits could be made by re-selling stock (direct investment through subscription) to others who saw value in a company. The secondary market, known also as the speculators market, came into prominence. This made the market more volatile. It was now fueled by highly subjective speculation about the individual company's future performance.

Despite the prominence of the NYSE (the NYSE remains highly regarded among stock markets because it only trades in the very large and well-established companies), other markets filled certain gaps for investor and company alike. Not all stock in the US was traded on the new NYSE; it was a membership-only enterprise. Such designation and the $25 membership fee kept many from trading on the exchange (Geisst 2004). However, the potential for profit did not deter non-member brokers from trading. Curbstone brokers as they became known, congregated outside the exchange and traded in stocks not covered on the NYSE, "and quickly developed a tradition that would lead to the organization of the New York Curb Market, the forerunner of the American Stock Exchange . . . The lack of a central location made the curb market the forerunner of the over-the-counter market as well" (Geisst 2004, p.21).

The NYSE required a minimum of 100 shares for a company to trade on its exchange. By the early 19^{th} century, many new companies sprang up surrounding the railroad and construction industries, many of whom could not meet the 100-share requirement. To meet the needs to raise capital and for a market to trade stocks that were not capable of NYSE listing, the curbstone brokers catered to the needs of these companies and traded stocks outside of the registered exchanges (Sobel 2000b). By the close of the Civil War (1861 – 1865), the textile, chemical, iron and steel, and even the oil industry were comprised of smaller companies that were first sold by the curbstone brokers. However, it was not until the early part of the 20th century that organization and standardization were brought to the curbstone brokers and their market.

In 1908, Emanuel Mendels established the New York Curb Market Agency to bring order to and codify trading requirements among the curbstone traders. This led to the establishment of the New York Curb Market in 1911 and to the recognition of broker and listing standards by way of a formal constitution (Sobel 2000a). It took until 1921 for the brokers to move from the curb to an indoor facility. In 1929, it changed its name to the New York Curb Exchange and became the leading international market listing more foreign issued stocks than any other US exchange. It became the American Stock Exchange (AMEX) in 1953, and in 2008 the merged with the NYSE to form a single exchange.

The NASDAQ: Founded in 1971 by the National Association of Securities Dealers, the National Association of Securities Dealers Automated Quotation, or NASDAQ, was the world's first electronic stock market. Its purpose was to increase stock trading that previously was only traded 'over-the-counter.' It functioned initially as a computer bulletin board with the purpose of lowering the spread (the difference between the bid and asking prices) and eliminating the profits made by brokerages from the spread and transaction float time. In 1988, NASDAQ began computer-assisted transactions that allowed investors to execute stock orders automatically. With the use of computerized trading (matching of buyers and sellers absent human involvement), the NASDAQ is the most efficient stock exchange in the world. In October 2004, the NASDAQ trading volume surpassed that of the NYSE (National Association of Securities Dealers 2007).

Today's Markets: Globally, the size of the stock market was estimated at about $36.6 trillion in October 2008. The total world derivatives market (exposure to asset-based derivative financial products) has been estimated at about $791 trillion, or 11 times the size of the entire world economy (CIA 2011). The value of the derivatives market far exceeds the securities market, because it is stated in terms of notional values (the nominal or face amount that is used to calculate payments made on the derivative instrument but not the amount that actually changes hands during a transaction (WebFinance 2011i)), so it cannot be directly compared to a stock or a fixed income security (a bond). Compounding this extreme value is that the majority of derivatives cancel each other out. Derivatives are typically a pairwise function: a derivative 'bet' on an event occurring is offset by a comparable 'bet' on the same event not occurring.

Stock markets now exist in virtually every developed nation and/or economy. The world's largest financial markets exist in the United States, United Kingdom, Japan, India, China, Canada, Germany, France, South Korea and the Netherlands (World Federation of Stock Exchanges 2012).

2.2.4 Stock Market Behavior

In its purest form, earnings are what drive the value or price of a stock (Maudlin 2004). However, the manner in which financial markets, and the stock market in particular, continuously change prices, they do not hold fast to this supposition. Adjusted for inflation, the earnings growth between 1965 and 1982 was approximately the same as that between 1982 and 1999. Yet, the stock market returns for those periods (as measured by the Standard and Poor's 500, the S&P 500) differed considerably: The former period had virtually no stock price growth while the later saw price growth over 1000%.

This marked difference is attributed to market conditions, market perception, and the behavior of market participants (investors). "[E]vidence demonstrates that share prices react to announcements about corporate control, regulatory policy economic conditions" (Cutler, Poterba, and Summers 1989, p.4).

Experience tells that the market deviates from its theoretical earnings-driven pricing structure and enters temporary periods or trends. Periods of slow, little, or no price growth (characterized as negative) are referred to as bear markets. Continually upward moving periods are defined as bull markets. Instances of overreaction occur, exemplified by Alan Greenspan (1996) a "irrational exuberance," the condition in which financial markets are experiencing a 'heightened state of speculative fervor' in which "news of price increases spurs investor enthusiasm, which spreads by psychological contagion from person to person ...despite doubts about the real value of an investment . . . drawn partly through envy and [that of] a gambler's excitement" (Shiller 2005, p.2).

The premise that historic behavior of the stock market is an indicator of its future performance has spawned many theories and hypothesis. Among them are the efficient-market hypothesis, groupthink, irrational behavior, and a number of empirical statistics-based formulas setting pricing and/or performance expectations for markets, classes or groups of shares, or individual share values.

Groupthink: Groupthink is psychological phenomenon occurring within groups when the desire for harmony in decision-making overrides a realistic appraisal of the alternatives. Whyte (1952) coined the term and further describes it as “rationalized conformity – an open, articulate philosophy which holds that group values are not only expedient but right and good as well” (Safire 2004). Of primary concern with groupthink is the loss of individual creativity, uniqueness, and independent thinking. Janis concluded while studying disastrous foreign policy events, e.g. Pearl Harbor, the Bay of Pigs, and the Vietnam War that “decisions were made largely due to groupthink, which prevented contradictory views from being expressed and subsequently evaluated” (Janis 1971). His characteristics of groupthink are summarized in Table 2.2.

Table: 2.2: Irving Janis’ Indicative Groupthink Symptoms and Practices (Kamau and Harorimana 2008)

Groupthink Symptoms	Groupthink Practices
Type I – Overestimations of the Group:	Incomplete Survey of Alternatives
Illusions of Invulnerability	Incomplete Survey of Objectives
Unquestioned Belief	Failure to Examine Risks of Preferred Choice
Type II – Closed-Mindedness	Failure to Reevaluate Previously Rejected Alternatives
Rationalizing Warnings	
Stereotyping Those Opposed	Poor Information Search
Type III – Uniformity Pressures:	Selection Bias in Collecting Information
Self-Censorship of Ideas	Failure to Work Out Contingency Plans
Illusions of Unanimity	
Direct Pressure to Conform	
Self-Appointed Mind Guards	

Groupthink provides the analytical foundation for the Greenspan irrational exuberance and forms one explanation for the difference between market performance and the base theoretical assumption that earnings drive prices. Simply put, investors have a propensity not to want to miss an opportunity and develop a herd mentality and follow a popular notion about market direction or about an individual stock. Conversely, investors do not want to be the last one out of the market when opportunity sours.

The Efficient-Market Hypothesis: The efficient-market hypothesis (EMH) is a three-part supposition that posits that financial markets are informationally efficient. The tenets of EMH are that above average rates of return are not possible in the long run given widespread information availability. It was developed by Eugene Farma as an academic concept in his PhD dissertation (Farma 1965).

In its weak form, the EMH speculates that the prices for stocks, bonds and/or property already account for all known and publicly available information. In the semi-strong form EMH speculates that prices reflect the information available under the weak form, and that the rice of a stock will instantly change when new information becomes publicly available. In the strong form, the EMH speculates that prices instantly reflect even insider information. While no empirical evidence is present to confirm EMH, it is widely held that financial markets are practically efficient in the presence of uncertainty (Desai 2011). Under EMH, any one person or all persons can be wrong about the market, but the market as a whole is always correct.

Irrational Behavior: Market movements do not always make rational sense. The market's response to new information can appear counterintuitive and impact specific securities opposite to that expected. This is attributed to the information being anticipated and predicted to meet certain thresholds. A market reaction may be counter to the information, should the information be better and/or worse than expected. Thus, financial markets and individual stocks and bonds can be influenced in either positive or negative directions by information releases, rumors, euphoria, panic, groupthink and/or Greenspan's irrational exuberance, though only in the short term.

Arbitrageurs (investors that attempt to profit from inefficiencies in the market by making simultaneous trades that offset each other to capture risk-free profits (WebFinance 2011a)) generally take advantage of such information asymmetry. Market behaviorists contend that investors behave irrationally when making investment decisions thereby incorrectly pricing securities. This in turn causes inefficiencies in the market and leads to opportunities to profit (Sergey 2008).

2.2.5 Capital Asset Pricing Model

The Capital Asset Pricing Model, or CAPM, was introduced independently by Jack Treynor (1961, 1962), William Sharpe (1964), John Lintner (1965), and Jan Mossin (1966), building on the earlier work of Harry Markowitz (1952) on diversification and modern portfolio theory. Sharpe, Markowitz, and Merton Miller (who built upon the theoretical work of Markowitz and Sharpe) shared the 1990 Alfred Nobel Memorial Prize in financial economics (The Royal Swedish Academy of Sciences 1990). It has been characterized as "one of the most important advances in financial economics" (Ross et al. 2005, p.295).

Within finance, the CAPM is used to determine a theoretically appropriate required rate of return of an asset, typically a stock, when the stock is to be added to an already well-diversified portfolio (a portfolio of stocks approximating the overall market risk). In its simplest form, the premise of the CAPM is that market participants need to be compensated in two ways: (1) for the time value of money, i.e. a dollar today is not the same as, nor can it be compared to, a dollar in the future and (2) for taking on added risk.

2.2.5.1 The CAPM Formula

The CAPM establishes a linear relationship between a stock portfolio's expected risk premium and the expected market risk premium. The CAPM formula (Black, Jensen, and Scholes 1972), which describes and quantifies this relationship and the expected return for an individual stock, is based upon the assumption that the expected return on the market is equal to the risk-free rate plus some compensation (premium) for the inherent market risk.

$$E(R_i) = r_f + \beta_i\left[E(R_m) - r_f\right] \qquad \text{[Eq. 2.1]}$$

where

$E(R_i)$ = the expected return on the capital asset (an individual stock)

$E(R_m)$ = the expected return of the overall market

$E(R_m) - r_f$ = known as the "market premium" or the "risk premium," it is the difference between the expected market rate of return and the risk-free rate of return

r_f = the risk-free rate of interest such as interest arising from government bonds

R_i = the return of an individual asset

R_m = the return of the overall market

β_i, = beta is the sensitivity of the expected excess asset returns to the expected excess market returns, where:

$$\beta_i = \frac{\text{Cov}\,(R_i, R_m)}{\text{Var}\,(R_m)} \qquad \text{[Eq. 2.2]}$$

The covariance (the statistics and probability theory measure and strength of the rate at which two random variables change together) of the return of the individual asset R_i to R_m, the return on the overall market *(Cov* (R_i, R_m)*)* is defined as:

$$\mathrm{Cov}\,(R_i, R_m) = \langle (R_i - \overline{R_i})(R_m - \overline{R_m}) \rangle \qquad \text{[Eq. 2.3]}$$

or explicitly as

$$\mathrm{Cov}\left(R_i, R_m\right) = \sum_{n=1}^{N} \frac{(R_i - \overline{R_i})(R_m - \overline{R_m})}{N} \qquad \text{[Eq. 2.4]}$$

And the variance (the statistics and probability theory measure of how far numbers spread and in particular how far they spread from the benchmark, the statistical mean or expected value; also referred to as σ^2) of R_m, the return of the overall market *(Var* (R_m)*)* is defined as:

$$\mathrm{Var}\,(R_m) = (R_m - \overline{R_m})^2 \qquad \text{[Eq. 2.5]}$$

or in explicit form as

$$\mathrm{Var}\,(R_m) = \sum_{n=1}^{N} \frac{(R_m - \overline{R_m})^2}{N} \qquad \text{[Eq. 2.6]}$$

where

R_i	=	the return of individual asset
$\overline{R_i}$	=	the asset benchmark, commonly the previous day's asset return
R_m	=	the return of the overall market
$\overline{R_m}$	=	the market benchmark, commonly the previous day's market return
N	=	the population size, the number of valuations being evaluated

The time value of money is represented by the risk-free rate (r_f). It compensates the investor for placing money in any investment over a period of time. The second component represents risk and identifies the compensation the investor needs for taking on additional risk through investment in the specific asset. This is calculated by taking a risk measure (beta) that compares the returns of the asset to the market over a period of time.

Figure 2.1: Analytic Breakdown of CAPM Equation (modeled after Ross et al. 2005)

Expected return on asset (individual stock)

Beta of individual asset

$$E(R_i) = r_f + \beta_i \bullet E(R_m) - r_f$$

Risk-free interest rate

Difference between the expected market return and the risk-free rate

The analysis depicted in Figure 2.1 implies that there is a linear relationship between the expected return on an asset and its beta. Historically, the market has an average rate of return greater than the average risk-free rate. Therefore, the market premium, $E(R_m) - r_f$, remains positive, and it is presumed that the expected rate of return for an individual asset (share of stock), $E(R_i)$, is positively related to its beta (Ross et al. 2005).

This can be demonstrated by two simple yet special cases: When $\beta = 0$ Eq. 2.1 yields $E(R_i) = r_f$. That is, the expected return on an asset with no apparent market risk is equal to the risk-free rate. Similarly, when $\beta = 1$ Eq. 2.1 yields $E(R_i) = E(R_m)$. This is where an individual asset apparent risk is equal to that of the greater market. Where the expected asset return is equal to that of the overall market.

2.2.5.2 Beta

Within market finance, beta (β) from financial portfolio theory represented by the CAPM is known to have several meanings. First, beta is a number describing the relation of an asset (an individual stock) or portfolio's relation of its returns with those of the financial market (Levinson 2009), where in practice the market as a whole is represented by the Standards and Poor's 500 Index. Beta is a key parameter, the seminal element of the CAPM. It is the measure of the volatility, or systematic risk, of an asset or portfolio in comparison to the risks within the market as a whole. Beta is also characterized as a measure of financial elasticity (an economic concept that is the measurement of the effects of changing one variable to the remaining others), relative volatility (the measure of price variation over time), diversifiable and systematic risk, and ultimately liquidity.

With its roots in regression analysis, an important factor when considering beta is that the average beta across an entire market, when weighted by the proportion of each asset's value to that of the portfolio, must equal 1.

$$\sum_{i=1}^{N} X_i \beta_i = 1 \qquad \text{[Eq. 2.7]}$$

where

X_i = the proportion of asset *i*'s market value to that of the entire market

N = the number of assets (individual stocks) in the market

β_i = as previously defined under Eqs. 2.1 and 2.2

Beta is simply characterized as "the influence the overall market's return on an individual stock" (Smith 2003b p.176). Beta is the asset-specific historic coefficient representing the degree to which an individual stock moves with the market. For example, a stock with a beta of 0.50 can be expected to move up or down about 50% as fast as the market. Conversely, a stock with a beta of 2.00 would be expected to move at twice the rate of the market. Betas of zero and less than zero are special cases, for which Table 2.3 further describes the meaning and range significance.

Table 2.3: Systematic Risk Classification by Beta Range (modeled after Gatfaoui 2010)

Beta (β) Range	Strategic Classification	Characteristic
$\beta > 1$	Offensive, Increased Risk	Typically cyclical assets experiencing market amplifying variations (market driving), e.g. growth stocks
$\beta = 1$	Moderately Defensive, Reduced Risk	Typically cyclical assets experiencing variations equal to the market (market mimicking)
$0 < \beta < 1$	Defensive, Minimal Risk	Typically non-cyclical assets experiencing less-than-market variation (market limiting), blue chip stocks
$\beta = 0$	Defensive, Zero Risk	Assets with no market correlation or dependency (market independent), e.g. cash, Ttreasury bonds
$-1 < \beta < 0$	Moderately Defensive, Increased Risk	Typically non-cyclical assets with inverse market variation (market 'safe havens'), e.g. utilities, dividend stocks
$\beta < -1$	Offensive, Increased Risk	Typically cyclical assets experiencing inverse market amplification variations (inverse market bets), e.g. hedge funds and derivatives
$\lvert \beta \rvert > 1$	Offensive, Heightened Risk	Typically any asset with higher risk than the overall market

2.2.5.3 CAPM Restrictive Assumptions

In his description of CAPM, Sharp (1963, 1964) illustrates how asset prices are established under conditions of market equilibrium (the state where market forces are balanced and no external influences and/or pressures will change asset values, i.e. the point at which the asset quantities demanded and supplied are equal (WebFinance 2011h)). However, market equilibrium is limited in its occurrence, otherwise prices would not fluctuate. Given this, the CAPM is subject to a set of restrictive assumptions (Gatfaoui 2010):

1. Perfect Markets: Markets operate in a perfect manner with no friction, tax or transaction costs, absent the potential for market manipulation – perfect competition.
2. Information Availability: Information is instantaneously and simultaneously available to all at no cost – perfect availability.
3. Investment Period: Participants uniformly invest for a single period of time.
4. Asset Configuration: Assets (individual stocks) are infinitely divisible and liquid.
5. Interest Rates: Interest rates are the same for borrowing and lending at the risk-free rate.
6. Rate of Return: Expected returns are normally distributed.
7. Investing Proclivities: Investors operate in a risk adverse and rational manner, expecting to maximize the future value of their investments.
8. Future Expectations: All investors maintain homogeneous forecasts about future interest rate variations.
9. Risk: Two distinct risk factors drive returns over time: asset-specific risk (idiosyncratic risk and/ or unsystematic risk, which is diversifiable) and overall market risk (systematic risk, which is nondiversifiable).

In practice, those assumptions do not hold for a variety of reasons, the foremost of which is that perfect markets, perfect information, and perfect competition rarely exist. Lending and borrowing rates differ in the marketplace. It is always more expensive to borrow than to lend, where lending is equated to investing (i.e. the difference between mortgage rates and savings deposit rates), the risk-free rate is not constant over time, transaction costs are a fact of market dealings, market returns are not necessarily normally distributed over the time period considered, and financial assets are not infinitely divisible. Trading remains fixed at specific quantities that are the required minimum for exchange.

2.2.5.4 Risk

Risks impact asset portfolios and individual assets in differing ways. Specific risk is peculiar to each asset and is not priced within the overall market. Conversely, systematic risk has no link to individual assets but is priced in overall market movements.

Specific Risk: When viewed exclusive to financial markets, specific risk (also referred to as unsystematic, idiosyncratic, or residual risk) is a diversifiable risk (one that by investing in a greater variety of individual stocks such that value of the individual assets does not move in synchrony with the overall market) that can be mitigated and/or eliminated. It is the individual asset based risk (company-specific or industry-specific risk) within a portfolio that remains uncorrelated with aggregate market returns. That is, the market risk has no effect on the risks found within each individual asset. When diversified through modern portfolio theory or MPT (Markowitz 1952), the risks average out or cancel out each other. MPT attempts to maximize the rate of return of a portfolio of a given risk level (specific to individual investor desires), or conversely and equivalently minimizing the risk associated with a desired rate of return. This is accomplished through careful selection and balancing of the respective proportions of individual assets with differing and opposite specific risks.

Systematic Risk: In finance, systematic risk, also referred to as market or aggregate risk, is a non-diversifiable risk. It cannot be eliminated by investing in a greater variety of individual assets (stocks). It is the risk that the value of an individual portfolio of assets will decrease due to risk factors specific to the nature and/or operation of the market as a whole. These risks include standard market factors: (1) equity risk, the risk that stocks and/or individual exchanges maintain an implied volatility and will generate aggregate price change; (2) interest rate risk, the risk that interest rates will fluctuate and/or become more volatile in their rate of change; (3) currency risk, the risk that the exchange prices for currency will increase or that their rate of change, their volatility, will increase; and (4) commodity risk, the potential for the price of commodity elements as previously defined to increase or for their availability to decrease.

Of particular concern is that diversification across an investment portfolio does not eliminate systematic risk. A portfolio holding all of the stocks in the market (S&P 500) in their respective weights does not overcome market risk, and when using the CAPM and MPT to determine expected rates of return, systematic risk is what plagues investors. Therefore, CAPM becomes the means to measure systematic risk (McClure 2010).

2.3 Analogy to Construction Network Schedules

2.3.1 Correlation of the CAPM to Network Project Schedule Systems and their Participants

Disparate entities, herein subcontractors, participate in a complex decision-making process subject to constraints and uncertainties as to whether a project will be on time. Where individual assets comprise a market, many activities and entities compose a project. Building upon financial portfolio theory, the CAPM typifies a possible determinant for the analogous behavior to those participating in construction projects and a potential element in the measure of a risk within network schedule uncertainties and their negative impacts.

Currently, float is the ubiquitous measure of flexibility that reduces risk and increases opportunity within construction projects governed by network schedule systems (Thompson and Lucko 2011). Its various types quantify the ability of an entire schedule or individual activities to accommodate uncertainty and absorb delays. The CAPM becomes *apropos* to schedule systems with beta being the measure of interaction within the systems and a measure of the need for flexibility and the expenditure of float. The Sharpe, Markowitz, Miller, et al. concept for the measure of the risk of an individual asset versus the movement of the overall market can be extended to the performance of an individual activity / entity and to an entire construction project.

Despite the apparent large difference between portfolio theory and project management practices, numerous conceptual analogies can be identified. Table 2.4 provides a comparison of their elements at varying levels of detail. A financial market, whose collective activities encompass multiple exchanges, investors, and individual assets, is analogous to a construction project's participants as defined by a network schedule system.

Table 2.4: Conceptual Comparison of Portfolio Theory and Project Management

Portfolio Theory Element	**Project Management Correlation**
Financial Market	Construction Industry
Specific Exchange	Construction Project
Portfolio	Collection of Activities / Subcontractors
Weight *(Asset Percentage)*	Value or Duration of Activity *(by a single subcontractor)*
Asset *(Individual Stocks)*	Subcontractors *(Different specialties of crafts and trades)*
Market Transaction *(Trade: buy / sell)*	Project Execution *(acquire, perform)*
Trade	Decision to Expend Float
Put Option	Sell Float
Call Option	Expend Float
Transaction Results	Activity / Subcontractor Performance
Dividend / Interest	Fee
Profit	Accretive Change Order
Loss	Deductive Change Order
Market Performance	Schedule Performance
In-the-Money	Ahead of As-Planned Schedule
Out-of-the-Money	Behind As-Planned Schedule
At-the-Money	On As-Planned Schedule
Market Mover	Critical Activity
Bull v. Bear Market	Over / Under Performing Activity
Systematic Risk	Project Risk
1. Equity Risk	1. Competitive Risk
2. Interest Rate Risk	2. Escalation Risk
3. Currency Risk	3. Labor Risk
4. Commodity Risk	4. Material Risk
Specific Risk	Activity Risk
Non-Market Moving *(Mitigated through Diversification)*	Non-Project Impacting *(Mitigated through Increased Resources)*

The interaction of non self-performed (outsourced) construction work planned and performed by different subcontractors correlates to the project and its network schedule just as individual assets (stocks, bonds, commodities, etc.) form an individual market or exchange.

While market interaction and the risk associated with performance can be measured by beta, no such measure exists with network schedule systems. Both financial markets and projects are uncertain and risky decision-making processes. Whereas individual assets have a quantifiable influence on the market via their price fluctuations, so do subcontractors have some as of yet undetermined influence on whether the project will be on schedule and within budget. This is the very definition of schedule risk: the potential and/or exposure to a loss or other consequences from a project or program not meeting its schedule

Hulett (2009) identified four problem categories of weaknesses within critical path method (CPM) scheduling:

1. Project scheduling is difficult. Projects vary in complexity, duration, and location, for which not all schedulers are appropriate for the specific task at hand.
2. The expectations / rules of scheduling are neither clear nor consistent. The approach to CPM schedule logic, constraints, resources, calendars, and activity durations vary between schedulers. Poor practices with these attributes can lead to unclear, imperfect, inadequate, or dangerous schedules.
3. Schedules are often asked to conform to unrealistic conditions. A scheduler is not permitted to produce a schedule that can be accomplished with available resources.
4. Project schedules are deterministic. Single activity durations are used that result in different paths / completion durations when uncertainty is considered.

2.3.2 Schedule Risk

Schedule risk begets questions about its materialization in the form of schedule slippages. With half a century of CPM scheduling in practice, questions about schedule overruns coalesce around three topics (Hulett 2009):

1. Can schedule overruns be predicted for individual projects to any degree of accuracy?
2. Is it possible to indentify the cause of schedule risk before it becomes problematic, thereby enabling project management to respond with appropriate risk mitigation methods and forestall any occurrence?
3. Can the universal causes of schedule slippages be determined?

Discussions of schedule risk and its derivative schedule risk assessment permeate the literature. As early as the 1970s, the U.S. Air Force recognized that schedule overruns were to be expected, as they were spawned by contemporary scheduling methods and not readily apparent:

> Initial cost and schedule estimates for major projects have invariably been over-optimistic. The risk that cost and schedule constraints will not be met cannot be determined if cost and schedule estimates are given in terms of single points rather than distributions ...A formal risk analysis is putting on the table those problems and fears which heretofore were recognized but intentionally hidden. (Lochry, 1971 p.91, 105)

Much like the Air Force's conclusions about schedules for their weapons programs, Nasir et al. (2003 p.518) describe construction projects as "complex in nature [with] many inherent uncertainties. These uncertainties are not only from the unique nature of the project but also from the diversity of resources and activities." They further conclude that there is a basic assumption for fixed project durations, but uncertainty abounds with the potential to cause slippages. However, it can be defined, classified, and addressed.

Table 2.5: Identification / Classification of Schedule Risks and Problems

McCabe 2003	Mulholland and Christian 1999	McConnell 1998 (partial list)
Area Conditions	**Nature of Construction Work**	**Schedule Creation**
site constraints	complex organizations	constraints dictated
work hour limitations	activity dependencies	best case expectation
traffic conditions	technique appropriateness	omits necessary tasks
Contractor	activity cost separation	not all members included
bondability	**Resource Allocation Process**	completion date advances
experience	network inadequacy	unforeseen delays
defective work / rework	uniform activity importance	**Management**
Design	single time value per activity	lacks sponsorship
fast track	allocation algorithm priority	staff reductions
complex / innovative	rules	budget reductions
quality / changes	limited resource types	centralized decisions
Environmental	resource / scope incongruities	emphasis on heroics
seasons	**Complex Operations/Schedule**	**Development Environment**
weather	schedules too complex to	facilities / tools unavailable
earthquake	institute	facilities / tools inadequate
Geotechnical	inappropriate network logic	**End Users**
archeological survey	models	changing requirements
localized conditions	specialized scheduling skill	late end-user involvement
unforeseen conditions	sets	**Customer**
Labor	excessive computer input /	unending review cycles
union disputes / strikes	output	poor communication
skill levels	**Lack of Schedule Confidence**	micro-management
availability / wage rates	model credibility for complex	unrealistic expectations
Materials	projects	**Contractors**
procurement challenges	lack of secondary business	late delivery
theft / fire	process	poor quality
storage / JIT delivery	**Activity Time Estimating**	**Product**
Owner	unreliable / unrealistic time	error prone
decision-making	estimates	unacceptable size / speed
financial stability	inexperience in activity type	not state of the art
payment frequency	marginally addressed schedule	**Personnel**
Political	risk	efficiency / productivity
community dissention		low motivation morale
delay by others		learning curve
directed stop		Critical skills unavailable
		Process
		cumbersome paperwork
		Too much / little formality
		Proj. mgmt. takes too long

Table 2.5 identifies several of the schedule risk variables or categorizations found in contemporary literature relative to construction projects (McCabe et al. 2003) and project management for other product / software development endeavors (McConnell 1998). Though apparently disparate in nature, the product / software development schedule risk elements have many common attributes, albeit described with different characterizations, syntax, and from a different perspective.

The causes or sources of schedule risk and overruns whether for weapon systems as in the case of the U.S. Air Force, complex construction projects per Nasir et al. or product development fit within three taxonomies: (1) schedule process problems (unique to the scheduling function and programs, i.e. CPM), (2) external challenges and/or constraints (specified completion dates, material / resource availability, etc.), and (3) human actions (productivity levels, interaction among participants, and differing levels of complexity / feasibility).

Browning, from a systems engineering perspective, focuses schedule risk at two levels: (1) the macro level, being project and/or product-centric, and (2) the micro level, process-centric (Browning 1998). Process risks or uncertainties are diagrammed within Figure 2.2, while the project / product-level risk is depicted in Table 2.6. Though product focused, Browning's categorizations can translate to a corresponding construction industry / project schedule risk concerns. Product performance risk translates to the project design (i.e. the design of a building, road, bridge, etc.) in the construction industry, development cost risk translates to project budget risk (i.e. construction cost and the potential for change orders, etc., or from a different perspective the adequacy of a project bid as it relates to the project scope), technology risk loosely translates to resource risk (i.e. process and/or equipment risk), and market risk translates to feasibility risk (i.e. the need for the road, bridge, building, etc., as designed and their adequacy to fulfill market expectations). Schedule risk equals schedule risk irrespective of the industry, process or circumstance being evaluated.

Table 2.6: Definitions of Categories of Product Development Risk (Browning 1998, Table 1, p.2)

Risk Category	Description
Product Performance Risk	Uncertainty in the ability of a design to meet desired quality criteria (along any one or more dimensions of merit, including price and timing) and the consequences thereof
Schedule Risk	Uncertainty in the ability of a project to develop an acceptable design (i.e. to sufficiently reduce performance risk) within a span of time and the consequences thereof
Development Cost Risk	Uncertainty in the ability of a project to develop an acceptable design (i.e. to sufficiently reduce performance risk) within a given budget and the consequences thereof
Technology Risk	A subset of performance risk, uncertainty in capability of technology to provide performance benefits (within cost and/or schedule expectations) and the consequence thereof
Market Risk	Uncertainty in the anticipated utility or value to the market of the chosen 'design-to' specification (including price and timing) and the consequences thereof

Browning's categorization spans the taxonomy's first two elements: process problems and external challenges. However, it is the schedule risk category and the consequences of its realization that remain of interest herein. Browning (1998) indentifies the following as consequences of deviant schedules through the realization of uncertainty:

- Rework adds additional time and money, consuming scarce resources
- Uncertainty mandates flexibility, requiring resource reserves

- There are no schedule absolutes with respect to time, money, and performance
- Uncertainty inhibits firm commitments for duration and/or completion
- Decision-making is ambiguous, which results in indecisiveness, hesitance abounds

Figure 2.2: Categories of Sources of Schedule Uncertainty (Browning 1998, Figure 1, p.2)

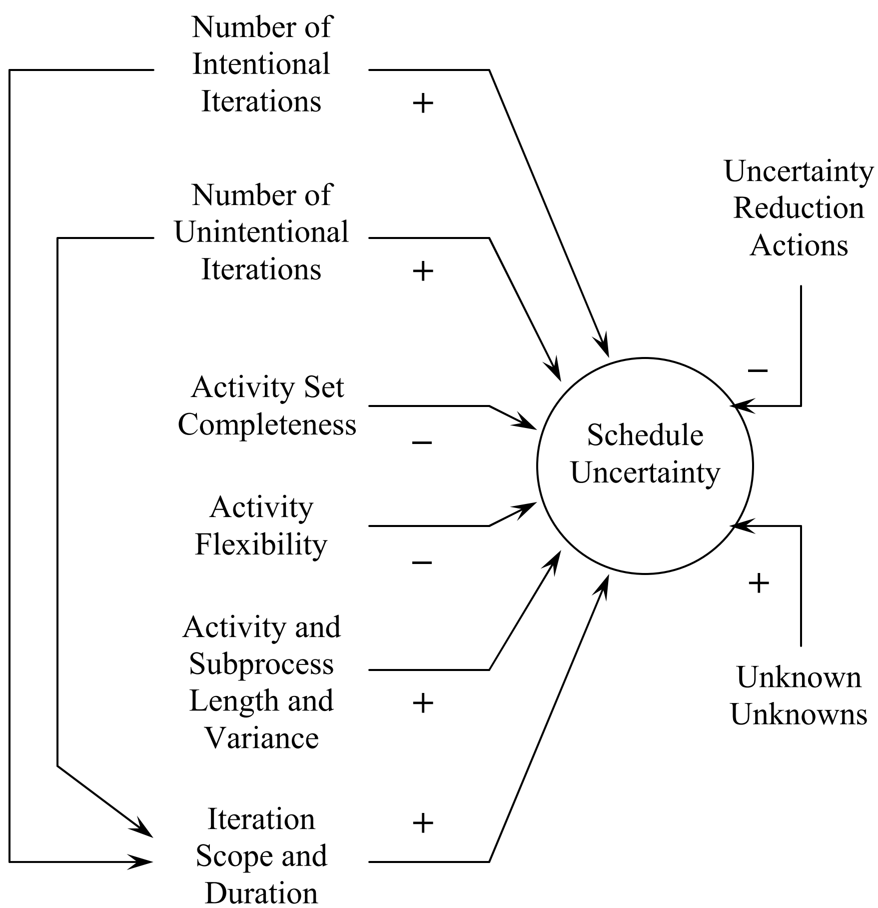


Figure 2.2 provides an overview of the process-centric drivers of schedule risk / uncertainty in the form of a causal loop with external influences. The positive (+) and negative (–) signs indicate the direction of the items influence on schedule risk / uncertainty.

That is, 'unknown unknowns' (with a positive sign) increase risk / uncertainty, while 'uncertainty reduction actions' (with a negative sign) decrease it.

The final element of the three-part taxonomy incorporates the human factor. Barseghyan (2009) contends that schedule risks are the result of human actions incurred over work durations and that "quantitative analysis of human action dynamics [herein considered as the interactions of subcontractors within individual projects] is becoming a problem of paramount importance for the challenges of project planning and schedule risk analysis" (Barseghyan 2009, p.1). He further contends that schedule risk problems are based on the "balance of human actions that incorporates effort, time duration, human productivity, size of action, and difficulty of task" (Barseghyan 2009, p.2). Barseghyan (2009, p.13) concludes that schedule risk analysis "is in crisis," because the old Gaussian approach is not adequate and a new human action based paradigm is required.

Group dynamics work, akin to groupthink, was used to identify shifts in risk that resulted in consensus decisions (Hopkinson 2001) but fail to reach the average initial option. It is suggested that group dynamics (peer pressure) can impact project inputs (e.g. schedule and/or development) that result in an overly optimistic bias.

Much has been written and researched with respect to the schedule process problems and external challenges and constraints. However, few consideration and measurement methodologies are available with respect to the human actions component of schedule risk and in particular the interaction among participants. It is here that this research is focused.

2.4 Schedule Risk / Float Interaction Measurement

The interactions of participants within construction network schedule systems, the subcontractors performing the many activities necessary to fulfill schedule and project expectations, parallel in concept that of the financial market. To translate financial portfolio theory and the application of beta, the measure of the relationship of individual asset performance to the performance of the overall market (or specific financial exchange), to network schedule systems and determine a method for defining the specific risk presented by the interaction among participants, and in particular critical activities, this research presents calculations, analysis, and conclusions by way of an exemplar.

2.4.1 Research Expectation

The overarching expectation is that this research depicts a method for the measurement, allocation and pricing of risk within network schedule systems as represented by the consumption of float that addresses the unique treatment and understanding of total float. Float is a vanishing commodity that it is generally consumed on a first-come, first-served basis and is not owned by any single entity (owner or contractor) or participant (subcontractors). More importantly, this segment of the research triplet seeks a method that defines an equitable means for measuring the risk that individual entities operating within the schedule systems and their interaction brings to the possibility of expending float by the participants most in need of its flexibility: critical network participants (those on the critical path who by definition have no float available).

2.4.2 Exemplar Development

2.4.2.1 Exemplar Foundation Elements

Reviewing the body of literature and analogous research, a simple network schedule used to depict network complexity and differing time calculation methods (Lucko 2005) is expanded to depict a project whose attributes and performance can easily, but accurately, portray the concepts under development and lend credibility to its analysis, conclusion(s), and extension.

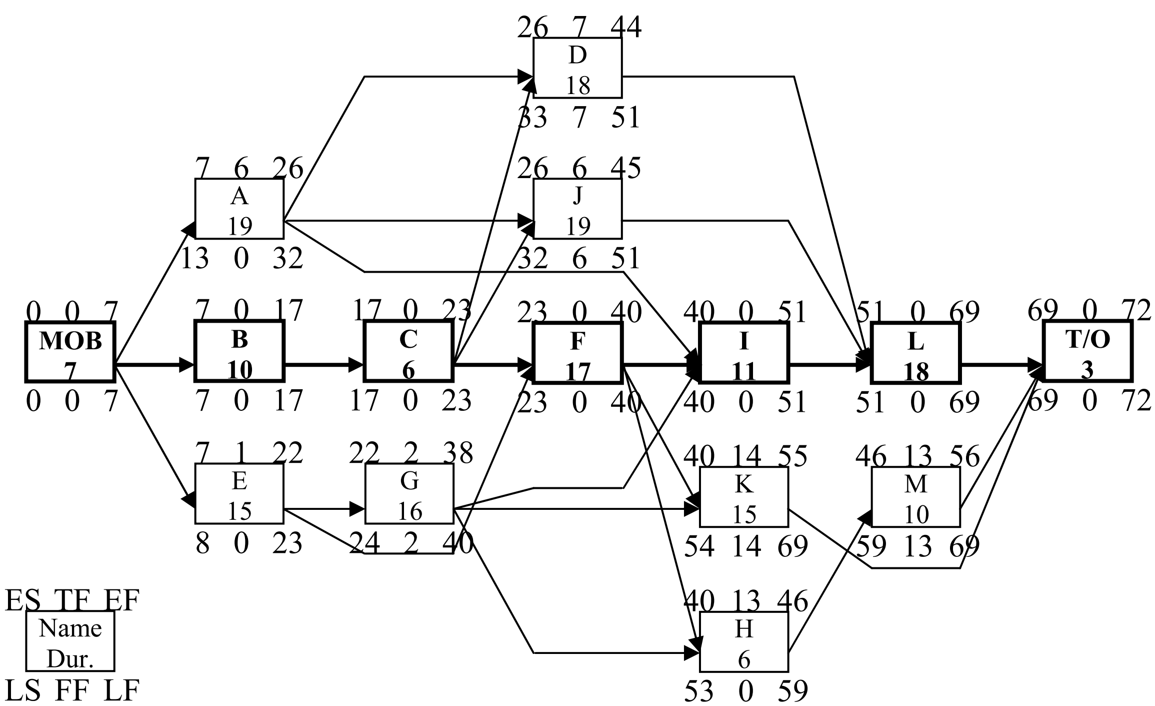
Table 2.7: Exemplar Inputs – Schedule Activity List with CPM Calculation Results

Activity	Duration (days)	Successor	Early Start (ES)	Late Start (LS)	Early Finish (EF)	Late Finish (LF)	Total Float (TF)
Mobilization	**7**	**A, B, E**	**0**	**0**	**7**	**7**	**0**
A	19	D, I, J	7	13	26	32	6
B	**10**	**C**	**7**	**7**	**17**	**17**	**0**
C	**6**	**D, F, J**	**17**	**17**	**23**	**23**	**0**
D	18	L	26	33	44	51	7
E	15	F, G	7	8	22	23	1
F	**17**	**H, I, K**	**23**	**23**	**40**	**40**	**0**
G	16	H, I, K	22	24	38	40	2
H	6	M	40	53	46	59	13
I	**11**	**L**	**40**	**40**	**51**	**51**	**0**
J	19	L	26	32	45	51	6
K	15	T/O	40	54	55	69	14
L	**18**	**T/O**	**51**	**51**	**69**	**69**	**0**
M	10	T/O	46	59	56	69	13
Turn Over	**3**	**N/A**	**69**	**69**	**72**	**72**	**0**
Total	72 days						

Boldface activities are on the critical path
Contractual Duration: 90 days
Adopted from Lucko (2005)

Table 2.7 summarizes the critical path elements found in the literature, and Figure 2.3 portrays the network logic, to which activity costs have been added representing those that could be expected of a small to mid-sized project of any type (construction-based projects not limiting). Beyond the schedule logic, further definition of project constraints is necessary for the exemplar to be sufficient for use herein and overcome an intrinsic shortcoming of a single critical path network schedule: the CPM duration is limited to the participants contained therein, not to other work performed by the subcontractors in fulfillment of other work / activities; nor are the actual durations taken to complete the work or the record for performance by the subcontractor contingent in fulfillment of the exemplar or other projects.

Figure 2.3: Exemplar CPM Network Schedule Diagram (Lucko 2005)



In addition, to further the exemplar to a financial portfolio theory approach to risk measurement, several other criteria must be defined, extrapolations made, and assumptions identified. Much like portfolio theory and the CAPM, the analysis requires two elements: a specific asset (herein an activity owner), and a market of exchange beyond the specific project / network schedule in which they operate and their respective performance measured against the benchmark to which they are gauged. That is, there is the need to define a larger body of work representative of similar projects and network systems in which there is common participation by the activity owner (subcontractors). This requires a broader array of schedule data representing other projects and actual durations.

To demonstrate the calculation of a theoretical beta within network schedule systems, the following additional information and assumptions are necessary. First, as-completed durations are needed for elements within the exemplar network system. To depict an appropriate cross section of critical activities, the performance of activities *B*, *F*, and *L* will be considered as they represent early, mid-point, and late critical activities respectively, as well as varying in duration (and assumed varying in cost, however cost parameters are not considered germane to this analogy). The as-completed durations for these activities are herein defined as 8 days for activity *B*, 25 days for activity *F*, and 18 days for activity *L*; with the overall exemplar system being completed in 90 days (72 days as-planned CPM activities plus 18 days of contract float (CF). Table 2.8 depicts the data array for eleven projects common to activities *B*, *F*, and *L*.

Table 2.8: Supplementary Exemplar Inputs – Additional Project Performance Data

	Project Performance		Activity B		Activity F		Activity L	
	Planned	**Actual**	**Planned**	**Actual**	**Planned**	**Actual**	**Planned**	**Actual**
Exemplar	72	90	10	8	17	25	18	18
1	180	185	25	17	30	36	45	45
2	60	62	10	4	10	8	20	15
3	45	60	5	6	10	15	15	12
4	15	21	8	4	15	21	5	12
5	365	350	45	30	72	70	120	115
6	220	280	20	25	60	75	80	60
7	90	105	20	20	15	21	20	21
8	75	75	15	15	15	14	15	10
9	120	135	40	35	30	45	32	25
10	270	260	45	50	75	70	90	75
11	30	45	5	2	10	15	10	10

2.4.2.2 Analogous Asset Rate of Return Definition

The CAPM formula (Eq. 2.1) identifies the rate of return on a capital asset (an individual stock) as R_i. To extend this concept to network schedule systems, identification of an analogous component is required. As the rate of return of an individual asset depicts the change in value over a given period (one day, one week, one month, etc.), the deviation from the as-planned duration for an activity or group of activities (depending upon the specifics being evaluated) is the corresponding 'rate of return' for network schedule systems. It will be designated as P_a for the schedule 'performance of an activity' (schedule participant / subcontractor).

2.4.2.3 Analogous Asset Benchmark Definition

The CAPM formula beta calculation (Eqs. 2.2, 2.3, and 2.4) identifies the benchmark for the rate of return on a capital asset (an individual stock) as $\overline{R_i}$. To extend this concept to network schedule systems identification, of an analogous component is required. As the benchmark for the rate of return of an individual asset is the previous asset price over the given period being evaluated, the as-planned duration for the activity is the corresponding 'benchmark' for network schedule systems. It will be designated as $\overline{P_a}$ for the schedule 'benchmark of an activity' (schedule participant / subcontractor).

2.4.2.4 Analogous Market Rate of Return Definition

The CAPM formula (Eq. 2.1) identifies the rate of return for the entire market (in practice, the S&P 500 index) as R_m. To extend this concept to network schedule systems, identification of an analogous component is required. As the rate of return for the market as a whole (or for a specific exchange) depicts the change in value over a given period (one day, one week, one month, etc.), the individual deviations from the as-planned duration for a collection of projects / network schedule systems (typically with activity owners within the given network system or project under consideration) is the corresponding 'market return' for the collection of projects represented by network schedule systems. It will be designated as P_c for the schedule 'performance of a cohort of projects.'

2.4.2.5 Analogous Market Benchmark Definition

The CAPM formula beta calculation (Eqs. 2.2, 2.3, and 2.4) identifies the benchmark for the rate of return for the entire market as $\overline{R_m}$. To extend this concept to network schedule systems, identification of an analogous component is required. As the benchmark for the rate of return of the entire market (in practice, the S&P 500 index) is the previous market rate of return over the given period being evaluated, the individual as-planned durations for a collection of projects / network schedule systems common to the activity owners within a given network system or project under study is the corresponding 'benchmark' for network schedule systems. It will be designated as $\overline{P_c}$ for the schedule 'benchmark of a cohort of projects.'

Table 2.9: Extended Exemplar Inputs – CAPM Beta Relative Elements

CAPM Beta Input	Variable	Descriptions		Variable	Schedule System Beta Input
Asset Rate of Return	R_i	Change in Asset Value from Benchmark	Actual Activity Performance	P_a	**Activity Performance**
Asset Return Benchmark	$\overline{R_i}$	Previous Period Change in Asset Value	Activity As-Planned Duration	$\overline{P_a}$	**Activity Performance Benchmark**
Market Rate of Return	R_m	Change in Market Value from Benchmark	Actual Project-Level Performance (for each Project Forming the Cohort)	P_c	**Project Cohort Performance**
Market Return Benchmark	$\overline{R_m}$	Previous Period Change in Market Value	Project-Level As-Planned Durations (for each Project Forming the Cohort)	$\overline{P_c}$	**Project Cohort Performance Benchmark**
Financial Beta Significance	β_i	Relationship of Asset's Returns to that of the Market's Returns	Relationship of Activity Performance (Duration) to that of the Cohort	β_c	**Schedule Beta Significance**

2.4.3 Schedule Risk via the CAPM Beta Analogy

2.4.3.1 Calculation / Equation Derivation

Calculating the beta, the relationship of the performance of a subcontractor (as represented by an activity within a network schedule system) in relation to the performance of the overall project against the as-planned durations, for the participants within construction projects can be accomplished via direct substitution of the variables identified in Table 2.9 into Eq. 2.2 as follows:

$$\beta_i = \frac{\text{Cov}\ (R_i, R_m)}{\text{Var}\ (R_m)} \text{ translates to } \frac{\text{Cov}\ (P_a, P_c)}{\text{Var}\ (P_c)} = \beta_c \qquad \text{[Eq. 2.8]}$$

Substituting Eqs. 2.4 and 2.6 into Eq. 2.8 yields the explicit form for the calculation of beta the participants of construction projects governed by network schedule systems:

$$\beta_c = \frac{\sum(P_a - \overline{P_a})(P_c - \overline{P_c})}{\sum(P_c - \overline{P_c})^2} \qquad \text{[Eq. 2.9]}$$

2.4.3.2 Network Schedule System Beta Development

Based on the exemplar inputs relative to actual schedule performance (the assigned durations for critical activities *B*, *F*, and *L*, and the cohort of other projects common to the selected critical activities), Table 2.10 contains the resulting deltas between actual project / activity performance and the benchmark as-planned durations and the resulting variance, covariances, betas, and the probabilities for finishing at or ahead of as-planned duration.

Table 2.10: Exemplar Calculations –
Delta Between Actual Duration and As-Planned

	Actual Performance v. Benchmark			
	Project $(P_c - \overline{P_c})$	**Activity B** $(P_a - \overline{P_a})$	**Activity F** $(P_a - \overline{P_a})$	**Activity L** $(P_a - \overline{P_a})$
Exemplar	18	-2	8	0
1	5	-8	6	0
2	2	-6	-2	- 5
3	15	1	5	-3
4	6	-4	6	7
5	-15	-15	-2	-5
6	60	5	15	-20
7	15	0	6	1
8	0	0	-1	- 5
9	15	-5	15	-7
10	-10	5	-5	-15
11	15	-3	5	0
Resulting Statistics				
Variance	353.73			
Covariance		49.50	88.58	-44.33
Beta *(β_c)*		**0.140**	**0.250**	**-0.125**
Performance Probabilities				
As-Planned	1	2	0	3
Over	9	3	8	2
Under	1	7	4	7
Within A-P Probability	0.250	0.250	0.333	0.833
Beyond A-P Probability	0.750	0.750	0.667	0.167

A-P = As-Planned Duration

2.4.3.3 Network Schedule System Beta Considerations

The calculation results in Table 2.10 depict beta results within expected parameters. That is, the network schedule system application of the beta concept is bounded by the upper limit of the overall project duration delta ($P_c - \overline{P_c}$), as logic holds that activity duration deviations cannot exceed that of the project to which it belongs. This differs from the financial portfolio theory application of beta that measures a rate of changes, the percentage-based rate of return metric, whereby a single element may have a rate of change greater than that of the collective market. The network schedule system application of beta (β_c) abides by the following constraints:

Theoretical Range for β_c: $-1.0 < \beta_c < 1.0$ [Eq. 2.10]

Expected / Practical Range for β_c: $-0.25 < \beta_c < 0.50$ [Eq. 2.11]

The theoretical range for β_c is bounded by 1.0 and –1.0 due the activity duration limitations that preclude the duration of an activity from exceeding the schedule to which it belongs, and that the as-planned versus actual durations are limited as follows:

$$\left(\left|P_a - \overline{P_a}\right|\right) \ngtr P_c \quad \text{and} \quad \overline{P_a} \ngtr \overline{P_c}$$

$$\text{except that} \quad \left(P_a - \overline{P_a}\right) \text{ may be} > (P_c - \overline{P_c}) \qquad \text{[Eq. 2.12]}$$

Similarly, the constraints of Eq. 2.7 hold for the aggregate of individual betas and their respective weights to that of the cohort.

The expected range for beta is limited based upon anecdotal experience of the author from a 25-year career in the AEC industry and a portfolio of constructed work in excess of $3 billion. It is maintained that projects typically finish within the bounds chosen.

2.4.3.4 Analysis of Exemplar Betas

Beta with respect to network schedule systems, β_c, is a measure of the magnitude and direction of an activity owner's aggregate performance with respect to the performance of an overall project cohort. The calculation results from Table 2.10 yield the following analysis.

Activity B, β_c = 0.140: An early activity within the exemplar network system and of relatively short duration yields a beta of positive value and diminutive magnitude. This characterizes the activity owner, the subcontractor, as presenting low specific risk for schedule delays that may impact the work of others and that of the overall schedule system. This can be attributed to being early work within the exemplar schedule (being less likely to experience delays due to the interaction of others and predecessor activity delays) and the portion of the overall schedule, the weight of the activity with respect to the remaining members of the network being relatively small (less significant) to the network system (as would most likely be within the rest of the project cohort).

When putting together a project schedule, the inclusion of the subcontractor representing exemplar activity *B* can be expected to present a schedule risk for extension, for delays beyond the as-planned duration, equal to 0.14 days for every day of overall project delay. The owner of activity *B* typically represents 14% of the schedule risk experienced within the project to which it is a party.

This early activity could be exemplified by the earthwork component of a construction project. It is an activity that does not significantly rely on other activities, is duration sensitive (time pressure abounds) due to the equipment-intensive nature of the work, and lacks sensitivity to material constraints.

Activity F, $\beta_c = 0.250$*:* A mid-point activity within the exemplar network system and of relatively medium duration yields a beta of positive value and notable magnitude. This characterizes the activity owner, the subcontractor, as presenting considerable specific risk for schedule delays that may impact the work of others and that of the overall schedule system. This can be attributed to being midway through schedule duration and being completed alongside the most other activities within the exemplar schedule (being more likely to experience delays due to the interaction of others and predecessor activity delays) and the portion of the overall schedule, the weight of the activity with respect to the remaining members of the network being relatively large (significant) to the network system (as would most likely be within the rest of the project cohort).

When putting together a project schedule, the inclusion of the subcontractor representing exemplar activity *F* can be expected to present a schedule risk for extension, for delays beyond the as-planned duration, equal to 0.25 days for every day of overall project delay. The owner of activity *F* typically represents 25% of the schedule risk experienced within the project to which it is a party.

This activity could be exemplified by the building envelope component of a construction project. It is an activity that is found in the middle of the activity sequence; relies on the predecessor work of other activities before commencing; is coordination, material and labor intensive; and has the potential to impact the work of parallel and successor activities.

Activity L, β_c = -0.125: A late or concluding activity within the exemplar network system and of relatively medium duration yields a beta of negative value and notable magnitude. This characterizes the activity owner, the subcontractor, as presenting little to no specific risk for schedule delays that may impact the work of others and that of the overall schedule system. In fact, the owner of activity *L* can be expected to routinely perform better than the as-planned schedule duration. This may be attributed to necessity as being one of the last activities to conclude the work, with the expectation of 'making up for past delays,' being completed alongside few other activities within the exemplar schedule (being completed independently and less likely to experience delays due to the interaction of others, but most probably experiencing the aggregate delays of predecessor activity). The portion of the overall schedule, the weight of the activity with respect to the remaining members of the network being relatively large (significant) to the network system (as would most likely be within the rest of the project cohort) also presents the best opportunity to perform better than the expected duration.

When putting together a project schedule, the inclusion of the subcontractor representing exemplar activity *L* can be expected to present little to no schedule risk for extension, for delays beyond the as-planned duration. It can be expected to reduce its as-planned duration equal to 0.125 days for every day of overall project delay. The owner of activity *F* typically represents no schedule risk experienced within the project to which it is a party, but rather is responsible for 12.5% of the schedule acceleration that may become necessary.

This activity could be exemplified by furniture, fixtures, and finishes or by systems furniture installation, one of the last activities undertaken in a building a construction project. It is highly repetitive, commences with the expectation that most all other activities have been completed (such that there is little interference) and has the potential to perform work with increased resources than planned and/or commence portions earlier than expected (pseudo-phasing of sorts). This provides for the ability to accelerate the schedule to 'make up' for previous delays.

2.5 Application

2.5.1 Construction Duration Variability

Construction projects, and in particular critical infrastructure projects, are notorious for time and cost overruns (Georgy et al. 2000, Kim 2007, Creedy et al. 2010, Shane et al. 2009). This is in part due to a "deficiency in managing the scope, time, quality, cost, [and] productivity" (Jergeas and Ruwanpura 2010, p.40). Depending on project type, schedule variation ranges from early finishes to far exceeding planned durations. In an international study of over 200 building projects, approximately one-third finished on or ahead of planned duration (Acharya et al. 2006), with 20% exceeding their planned duration by more than 50% (Table 2.11 depicts the full schedule variability findings of Acharya et al.). Bhargava et al. (2010) found that for about 90% of projects in a 1,800-plus highway construction project study, the actual construction durations exceeded the planned construction duration. In some instances actual construction duration exceeded that expected by a factor of five. Table 2.12 identifies the full array of schedule durations.

Table 2.11: Building Project Duration Delay (Acharya et al. 2006, Table 4)

Schedule Delay	Percent of Projects
Early Finish	5.5%
No Delay	28.9%
Under 10%	7.0%
11 - 25%	14.8%
25-50%	23.4%
Over 50%	20.3%

Table 2.12: Highway Construction Project Durations (Bhargava et al. 2010)

Percent of As-Planned Schedule	Percent of Projects
Early Finish	11.12%
100 - 200%	43.29%
201 - 300%	16.70%
301 - 500%	19.28%
501 - 1000%	8.27%
Over 1000%	1.34%

Project management is doubly challenged, as “poor scheduling and control” are contributing causes for these overruns (Akpan and Igwe 2001, p.367, Jergeas and Ruwanpura 2010), and then managers remain challenged to mitigate their combined impact. The introduction of a risk measurement model represents a vehicle to overcome these challenges and measure the risk presented by the interaction of the critical activities of a network schedule system. To demonstrate this, duration variation is applied to the exemplar via Monte Carlo simulation; the results of which will form the data population for statistical analysis to support the hypothesis of the exemplar.

2.5.2 Probability Distribution Function Selection

Within construction industry simulation modeling, much consideration has been given to the seminal element, the appropriate probability distribution formula (PDF). Arízaga (2007) determined that a multiplicity of PDFs are suitable for use in construction industry modeling. They include the normal and lognormal distributions (Touran 1997), the beta distribution (Touran 1997, Fente et al. 2000, Maio et al. 2000, Schexnayder et al. 2005) (the beta distribution is not to be confused with the portfolio theory beta and its extension considered herein), and the triangular distribution (Back et al. 2000, Arízaga 2007). Wilson et al. (1982) studied the use of beta versus triangular distributions on ground operations, concluding that there were not significant differences in the simulation outputs.

Raymond noted that risk implies a stochastic (probabilistic) process whose quantification is uniquely adaptable to modeling by the application of a PDF to each element in a cost estimate or schedule (Raymond 1999). When applying Monte Carlo techniques, Raymond concluded that:

> A simple triangular distribution is a reasonable PDF for describing risk or the uncertainty for a cost element or task duration estimate. Its structure is based on the minimum possible cost and duration (plan best case), the most likely cost and duration (budget most likely) and the maximum possible cost and duration (project worst case)… The parameters are simple, intuitively easy to comprehend, and amenable to a mathematical formulation comparable with cost and schedule models and fast Monte Carlo analysis. Other more complex distributions could be used such as the Beta or Weibull, but little if anything is gained, and the intuitive simplicity of the triangular distribution is lost (Raymond 1999, p.148).

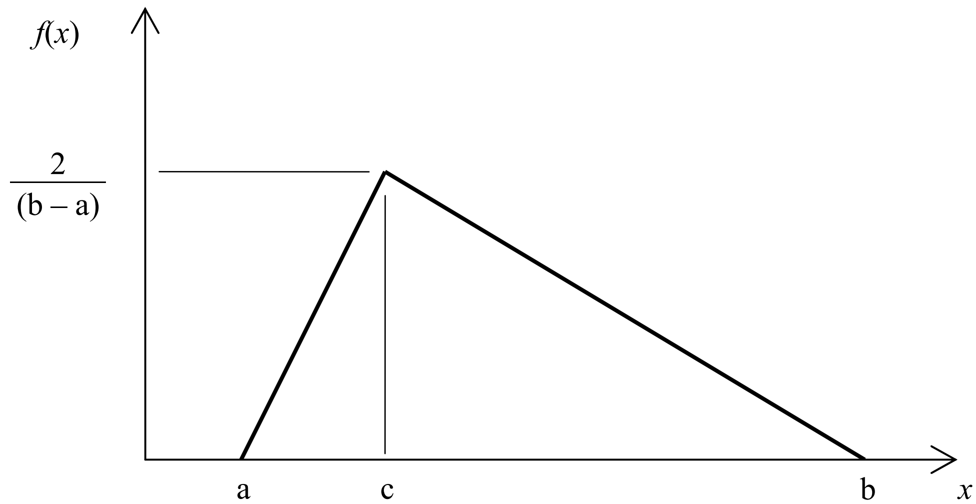
2.5.3 Triangular Distribution

In probability theory and statistics, the continuous triangular probability distribution is defined by three points per Eq. 2.13 and Figure 2.4: (1) the minimum value *a*, (2) the most likely value or statistical mode *c*, and (3) the maximum value *b*. The direction of the skew of the triangular distribution is set by the size of the most likely value relative to the minimum and the maximum. It is perhaps the most readily understandable and pragmatic distribution for basic risk modeling and has a number of desirable properties including: a simple set of parameters, the use of a modal value (i.e. the most likely case), and a deterministic probability distribution generated by the range of possible values.

$$f(x|a,b,c)=\begin{cases} 0 & x < a \\ \dfrac{2(x\text{-}a)}{(b\text{-}a)(c\text{-}a)} & a \le x \le c \\ \dfrac{2}{(b\text{-}a)} & x = c \\ \dfrac{2(b\text{-}x)}{(b\text{-}a)(b\text{-}c)} & c < x \le b \\ 0 & x > b \end{cases}$$

[Eq. 2.13]

Figure 2.4: Graphical Representation of Triangular Distribution



Conversely, the triangular distribution has two main disadvantages. First, when the parameters result in a skewed distribution (the area of the triangle being predominantly on one side of the mode value), then there may be an over-emphasis of the outcomes in the direction of the skew. Second, the distribution is bounded on both sides, whereas many real-life processes are bounded on one side but remain unbounded on the other.

2.5.4 Exemplar Distribution Calculations

Beyond selection of the PDF, herein the triangular distribution function, additional constraints are required to implement a Monte Carlo simulation of the exemplar schedule network. Of primary concern are the *a* and *b* components required for the distribution, the lower and upper bounds of the distribution respectively, with *c*, the mode value, herein defined as the as-planned or expected duration.

The best depiction of the performance of construction projects relative to schedule is represented by the Acharya et al. (2006) study. It was shown that approximately one-third of the projects evaluated finished on or ahead of planned duration, with 20% exceeding their planned duration by more than 50%.

Building upon the Acharya et al. (2006) findings and setting a definitive expectation for the performance of a simulation, the variability of individual activity durations will be established to fit the one-third – two-thirds distribution of projects finishing at or ahead of as-planned duration versus those finishing behind as-planned duration respectively. Applying this to the exemplar durations, the *a* and *b* components can be determined as a percentage of each mode value *c*. Table 2.13 depicts the confirmation process for potential values associated with the Acharya et al. project durations and that are to be used within the simulation for *a* and *b*, and the probability for occurrence of the mode value *c*, as calculated by the formula:

$$P(x|c): \frac{2}{(b-a)} \qquad \text{[Eq. 2.14]}$$

When calculating the upper and lower bounds for the exemplar PDF values (from Table 2.13), rounding must occur, because partial days are anathema to network schedule systems. Herein when evaluating integer rounding functions to arrive at whole days, numeric (decimal) rounding rules were employed to zero decimal places. Similar constraints regarding integer representation of activity durations (i.e. rounding to zero decimal places) are required for the triangular PDF within the Monte Carlo simulation software, because the triangular distribution is a continuous function for which any value between the *a* and *b* limits is possible.

2.5.5 Monte Carlo Simulation

To provide further insight into the interactions of the exemplar schedule participants under the aforementioned conditions of uncertainty and variability, a Monte Carlo simulation (or simulation) will be performed. Absent a vast array of network schedules with common participants (activity ownership) against which to compare as-planned versus actual durations and calculated beta, a method to approximate a cohort of projects (to simulate the marketplace) is to apply PDF constraints to the exemplar schedule for which multiple iterations are then produced and analyzed as a bootstrapped cohort of projects.

2.5.6 Statistical Bootstrapping through Monte Carlo Simulation

The bootstrap method is most useful when the sample size is insufficient for straightforward statistical inference. It is generally useful for estimating the distribution of a statistic (e.g. the mean, variance, etc.) when normal theory is unavailable to help estimate the statistical distribution, the statistical distribution herein being the resulting schedule duration.

> So suppose that we have just one sample. Is there any way to use that one sample to compute an estimate of the sampling distribution of a statistic? This is where the bootstrap comes in. The idea is to repeatedly sample (with replacement) from the single sample you have, and use these "samples" to compute the distribution of the statistic in which you are interested. [By way of a] Monte Carlo exercise, we drew a "fresh" sample each time…from the single sample that we have." (Varian 2005, p.772)

This approach to statistical population generation is a form of 'bootstrapping' data. To bootstrap data, the "data-based simulation method for statistical inference" (Efron and Tibshirani 1993, p.5), is to create an initial population of data (herein the as-planned network schedule) and then through drawing and replacement (herein individual iterations of the simulation model) a bootstrapped population of data (herein the marketplace) is derived. "…[R]epeat this process a large number of times, say 1000 times, to obtain a 1,000-bootstrap replica [of the population]" (Efron and Tibshirani 1993, p.5). "The particular goal of bootstrap theory is a computer-based implementation of basic statistical concepts" (Efron and Tibshirani 1993, p.6).

Efron and Tibshirani (1993, p.52) state that "[a]pproximations obtained by random sampling or simulation are called Monte Carlo estimates. …[C]omputer methods other than straightforward simulation can sometimes reduce manyfold the number of replications needed to obtain a prespecified accuracy."

The purpose of using Monte Carlo simulation is to bring variability and uncertainty to the activity durations within the exemplar network schedule system and gauge the performance of the activities forming the critical path with respect to that of the entire schedule system. The activity duration population generated by the simulation model will serve as the bootstrapped data for the calculation of beta for activities *B*, *F*, and *M* using Equations 2.8 and 2.9. It is expected that this will provide insight into the suppositions drawn from the exemplar hypothetical schedule population.

2.5.7 Monte Carlo Simulation Model Development

The Monte Carlo simulation model used to bootstrap a statistical population was developed using the @Risk™ (At-Risk) Risk Analysis and Simulation Add-In Program, Version 5.7 (September, 2010) for Microsoft® Excel (2007), from the Palisade Corporation, Ithaca, NY.

To create the working model representing the exemplar network schedule system, an activity data and calculation box representing the schedule components for each activity was developed and positioned within the Excel worksheet in relative position to that depicted in the exemplar CPM network diagram (Figure 2.3). Lines with arrows were added to depict the schedule logic, while simple Excel "MAX" statements set the predecessor to successor logic for the early start (ES) of each activity (with the maximum being the maximum schedule duration taken by the predecessor activities requiring completion before the successor may begin) . Early finish (EF) duration is calculated by adding the duration of the subject activity to the early start. This is completed for all activities such that the schedule forward pass is complete. A backward pass is then completed using only "EQUAL" (=) statements from activity to activity upon subtraction of the activity duration from the late finish (LF) to create the early finish (EF). A portion of the model is depicted in Figure 2.5. The activity and data and calculation box for activity *L* is deconstructed in Figure 2.6, including the @Risk PDF parameters and output expectations.

Figure 2.5: Excel-Based @Risk Simulation Model – Partial Segment

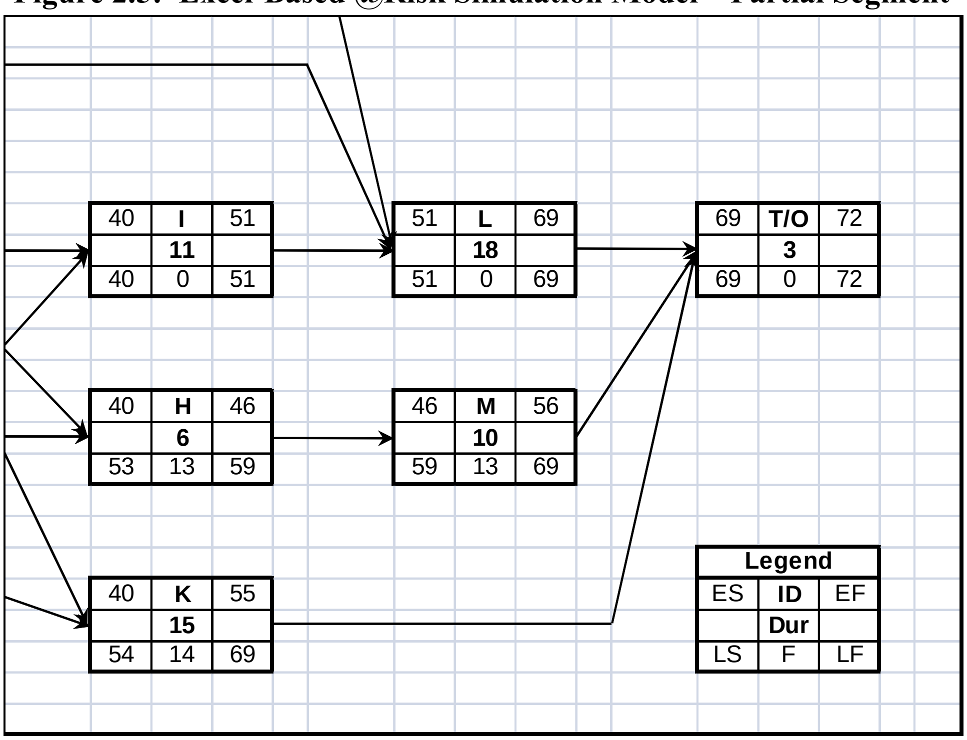
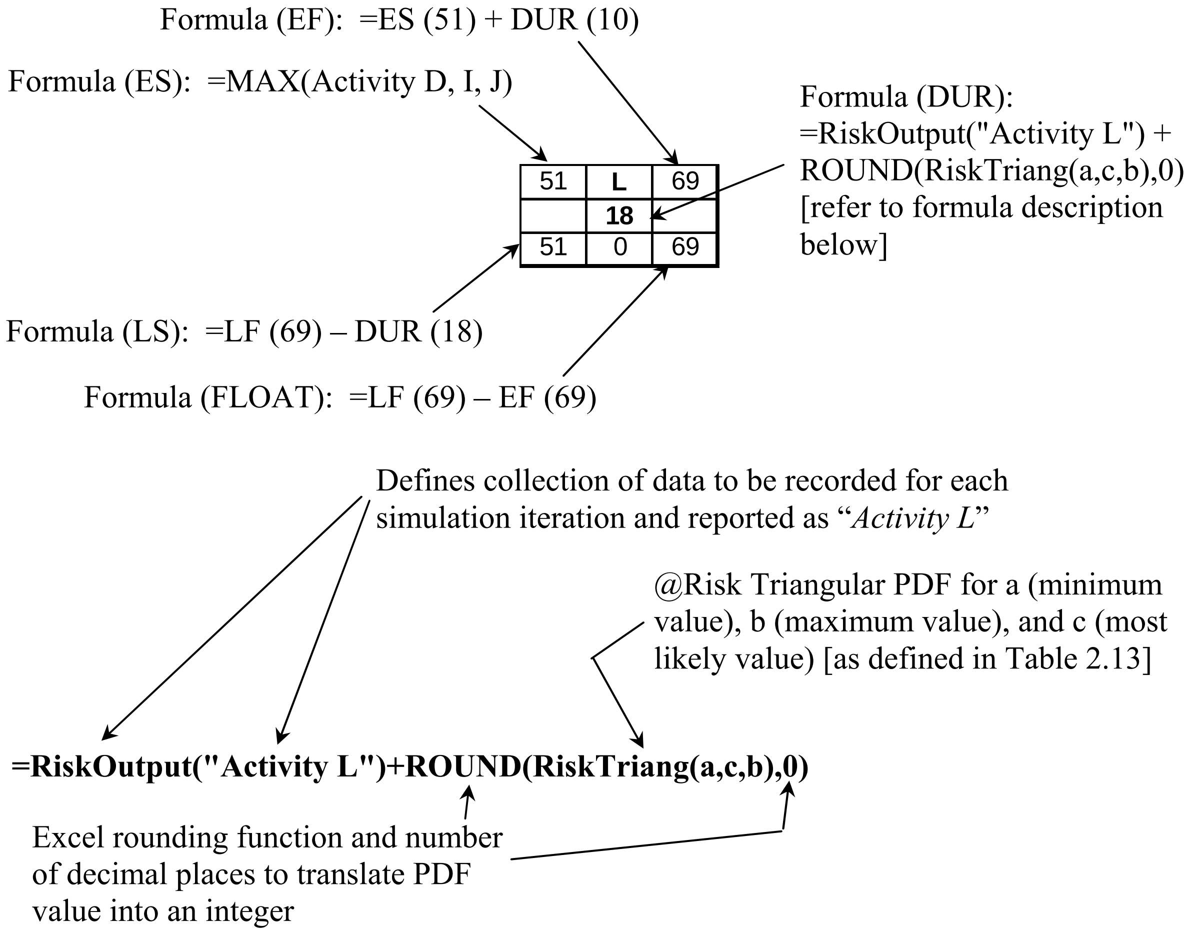


Figure 2.6: Data and Calculation Box Deconstruction – Activity *L*



2.5.8 Triangular Probability Distribution Function Development

To arrive at the baseline PDF to be used in the Monte Carlo simulation, activity durations were varied as a percent function of the as-planned duration and analyzed against the idealized one-third – two-thirds Acharya et al. (2006) study distribution for schedule duration distribution versus the 72-day expected duration (the as-planned duration not including the 18 days of contract float (CF)). To calibrate the model, that is to set the PDF for activity level performance, the individual as-planned durations were translated into a triangular PDF by multiplying the as-planned duration by one-plus or one-minus a percentage for the *b* and *a*

PDF elements respectively (e.g. the 10% / 20% activity duration simulation results per Table 2.14) were created by multiplying each as-planned duration by 1-10% for the lower duration and 1+20% for the upper duration). The resulting durations were rounded to become integers (zero decimal places) to represent whole days. This process was repeated at varying percentages in five percent increments until the simulation results converged near the desired duration distribution, and then evaluated at one percent and one half of one percent increments to determine the best fit. The Excel table used to generate the PDF values and populate the duration values for each activity data and calculation box used in the ranges necessary to calibrate the simulation model is depicted in Table 2.13, where *a* = minimum value, *b* = maximum value, and *c* = most likely value.

Table 2.13: Monte Carlo Simulation Activity Duration PDF Generator

Triangular PDF			
Percent < APD			10%
Percent > APD			20.0%
ID	**a**	**c**	**b**
Mob	6	*7*	8
A	17	*19*	23
B	9	*10*	12
C	5	*6*	7
D	16	*18*	22
E	14	*15*	18
F	15	*17*	20
G	14	*16*	19
H	5	*6*	7
I	10	*11*	13
J	17	*19*	23
K	14	*15*	18
L	16	*18*	22
M	9	*10*	12
T/O	3	*3*	4

APD = As-Planned Duration

2.5.9 Simulation Results

The results for the simulations depict the same three activities previously exemplified, their resulting betas, and are presented in Table 2.14. Based on these results, the overall project duration converges at the desired distribution between the 25% / 25% version and the 25% / 30% version. Applying finer granularity to the percentages, a best fit appears at the 25% / 28.5% distribution. A 100,000 iteration simulation was run to provide a sufficiently large statistical population.

Table 2.14: Simulation Model Combined Calibration and Validation Results

Model Calibration			Model Validation						
Activity Durations			Performance (72-day As-Planned Duration)				Beta (β_c) for Activities		
Less than APD*	Greater than APD	Model Result Type	Less than APD	Greater than APD	Max (days)	Min (days)	Activity B (10 days)	Activity F (17 days)	Activity M (18 days)
10%	25%	Average	5.40%	94.42%	84.6	69.6	0.102	0.232	0.370
15%	50%	Average	0.36%	99.64%	95.6	70.6	0.055	0.263	0.362
20%	25%	Average	24.58%	75.42%	83.4	66.8	0.096	0.218	0.419
25%	25%	Average	38.00%	62.00%	82.8	64.4	0.083	0.243	0.362
25%	28%	Average	35.00%	64.58%	84.0	64.6	0.082	0.291	0.340
25%	**28.5%**	**100,000**	**33.50%**	**66.50%**	**87.0**	**61.0**	**0.078**	**0.281**	**0.344**
25%	29%	Average	32.68%	67.46%	84.0	64.6	0.075	0.270	0.357
25%	30%	Average	31.26%	68.74%	84.2	64.6	0.068	0.279	0.352
25%	50%	Average	3.68%	96.32%	96.8	66.8	0.060	0.264	0.346
30%	30%	Average	41.46%	58.54%	83.2	61.6	0.067	0.263	0.371
50%	50%	Average	26.72%	73.28%	94.4	59.0	0.049	0.237	0.418
90%**	100%	Average	16.72%	83.28%	115.2	54.2	0.047	0.204	0.443
Intuition Per Table 2.14		Average	31.04%	68.96%	88.8	62.2	0.147	0.319	0.038
Intuition Per Table 2.15		**100,000**	**30.50%**	**69.50%**	**93.0**	**59.0**	**0.134**	**0.329**	**0.032**

Model result are the average of five 1,000-iteration simulations unless noted otherwise

* APD = As-Planned Duration

** Less than APD percentages become problematic above 66%, as activity *T/O* rounds to zero. This requires an override via the establishment of a 1-day minimum duration.

Similar to the calibrated distributions, an intuition-based version is offered. In this version, the activity durations are based upon an intuitive approach for activity level duration variation from that planned and consider elements such as sequence in the schedule, the level of parallel activity, comparative duration, and the potential for acceleration as depicted in Table 2.15. The skew and potential for acceleration are dependent upon the activity characteristics as depicted in the exemplar CPM Network diagram (Figure 2.3).

Table 2.15: Intuition-Based Simulation PDF Values and Characterizations

Probability Distribution Function Values					Skew			Position			Parallel Activity		Duration			Probability for Success			Acceleration Potential	
ID	**a**	**c**	**b**	**P(x|c)**	**Left**	**None**	**Right**	**Early**	**Middle**	**Later**	**High**	**Low**	**Short**	**Medium**	**Long**	**High**	**Medium**	**Low**	**Yes**	**No**
Mob	3	7	7	0.500	■			■				■	■			■			■	
A	14	19	21	0.286	■			■			■				■		■		■	
B	6	10	18	0.167			■	■			■			■		■				■
C	3	6	9	0.333		■		■			■		■			■				■
D	13	18	29	0.125			■		■		■				■			■		■
E	12	15	24	0.167			■		■		■				■		■			■
F	10	17	26	0.125			■		■		■				■		■			■
G	11	16	26	0.133			■		■			■			■	■				■
H	3	6	11	0.250			■		■		■		■				■			■
I	5	11	17	0.167		■			■		■				■			■	■	
J	13	19	30	0.118			■		■		■				■			■		■
K	11	15	24	0.154		■			■		■				■		■		■	
L	15	18	18	0.667	■					■		■			■	■			■	
M	8	10	14	0.333			■			■		■		■		■				■
T/O	2	3	5	0.667			■			■		■	■			■				■

2.5.10 Analysis of Simulation Betas

Beta (β_c) for the simulated project cohort yields results in values within hypothesized ranges presented within the exemplar cohort. That is, early activities with short durations are relegated to the lower end of the beta range, while longer durations and later in the schedule fill out the higher beta range (this is contrary to the exemplar beta for activity *M* which was crafted to be an acceleration activity with a negative correlation to the exemplar cohort).

As expected, when activity durations become more volatile, when the PDF ranges presented in Table 2.14 approach 100% (i.e. the 90% / 100% variation), the most extreme betas result. For activity *M*, a late critical activity of substantial duration, beta reaches the highest simulated value at 0.443, and for early critical activity *B* it yields the lowest value at 0.047. Beta for activity *F*, a critical activity in the middle of the schedule falls in between at 0.204.

The betas for the range of activity durations depicted in Table 2.14 increase as the PDF range increases, the simulation of increased schedule volatility and uncertainty. It mimics the supply chain 'bullwhip effect' phenomenon (also known as the 'whiplash' effect) (Forrester 1961) wherein as demand changes in forecast-driven distribution channels become more volatile, the corresponding swings in inventories become larger and larger. This is akin to the amplitude oscillation increases along the length of a bullwhip produced by small movements at the handle and resulting in large swings at the unencumbered end (Forrester 1961, Lee et al. 1997, Mason-Jones and Towill 2000).

The bullwhip effect occurs when demand order variability in the supply chain becomes amplified as it moved up the supply chain (when applied to the network schedule systems this is the amplification of predecessor delays on successor activities and overall performance). Distorted information from one end of a supply chain to the other can lead to tremendous inefficiencies (i.e. schedule delays and deviation from the perfect schedule).

With respect to the expected performance of the simulated project cohort (i.e. the PDF values delivering the desired schedule duration at the Acharya et al. one-third – two-thirds distribution), the 100,000-iteration simulation at the 25% / 28.5% variation represents the most probable beta values: activity *B* at 0.078, activity *F* at 0.231, and activity *M* at 0.344. Following suit with the analysis of the hypothetical exemplar cohort, activity *B* represents a low risk and can be expected to be responsible for approximately 8% of the schedule delays on the projects in which it participates. Likewise, activities *F* and *M* can be expected to produce 23% and 34% of the schedule delays respectively.

Of particular note with respect to the results for the simulation cohort is that only positive betas were generated. As the preponderance of schedule performance was beyond the as-planned 72-day duration (exclusive of the 18 days of contract float) and all activities had triangular PDF values skewed to the right, it was impossible for any activity to negatively correlate to the performance of the schedule cohort.

Conversely, the Intuitive simulation model is designed to approach activity durations based on experience, insight, and knowledge into the inner workings of construction projects. A key element is that when acceleration of an activity based on either little parallel activity (the situation in which little or no other work is scheduled to take place) or based upon the overall need of the project (typically experienced by late activities and the need to 'catch up' to the as planned overall duration due to predecessor activity overruns) is needed or possible, the PDF values are set to routinely implement shorter durations. This is accomplished by shifting the PDF from being right skewed (where $b - c > c - a$) to having no skew (where $b - c = c - a$) or being left skewed (where $b - c \leq c - a$). Activities meeting this requirement are included in Table 2.16.

Table 2.16: Intuition-Based Simulation Activities with No Skew or Left Skew

ID	a	c	b	P(x|c)	Skew Value*	Justification / Commentary
Mob	3	7	7	0.500	Left 4 days	Capable of preplanning for success, no dependence on predecessor activity
A	14	19	21	0.286	Left 3 days	Initial lengthy activity not on critical path, not impacted by nor dependent upon predecessor work
C	3	6	9	0.333	No Skew	Early critical activity with little parallel activity and short predecessor activity
I	5	11	17	0.167	No Skew	Late critical activity positioned to catch-up for delays previously experienced
K	11	15	24	0.154	No Skew	Lengthy late activity not on critical path, expected to be capable of acceleration
L	15	18	18	0.667	Left 3 days	Penultimate critical activity, last significant opportunity for catch-up

*Skew Value = $(b - c) - (c - a)$

The results of the 100,000-iteration simulation produced an overall schedule duration distribution of 30.5% ahead of as-planned duration and 59.5% greater than as-planned duration, falling short of the Acharya et al. one-third / two-thirds expectation, but approximately within 10% in either direction. Conversely, the Intuitive simulation produced a greater range of overall schedule durations at 34 days (from 59 days to 93 days) than that of the standard 100,000-iteration simulation at 23 days (from 61 days to 87 days).

However, comparing betas for the 100,000-iteration simulations (Table 2.17), one difference becomes apparent, which lends support to the supposition of the hypothetical exemplar project cohort results. While not negative (as in the hypothetical exemplar cohort), the beta for activity *M* becomes extremely small.

Table 2.17: 100,000-Iteration Simulation Beta (β_c) Comparison

Simulation Version	Activity *B*	Activity *F*	Activity *M*
Hypothetical Exemplar Cohort*	0.140	0.250	-0.125
25% / 28.5%**	0.078	0.281	0.344
Intuitive**	0.134	0.329	0.032

* Beta values in Table 2.10
** Beta values in Table 2.13

Negative betas, and in the case of the activity *M* of the Intuitive model an extremely small beta, depict activity performance opposite to schedule delays. They dampen the bullwhip effect and reduce schedule delays in the network schedule system. Lee et al. (1997) identify four causes of the bullwhip effect, for which corresponding construction project network schedule systems elements are presented in Table 2.18.

Table 2.18: Conceptual Comparison of Supply Chain Bullwhip Effect and Project Management

Supply Chain Bullwhip Cause	Project Management Correlation
Imperfect (Unstable) Demand Forecasting	Unrealistic / Inadequate Initial Planning / Resulting Schedule Duration(s)
Large / Interrupted / Inconsistent Order (Batch) Size	Insufficient Schedule Logic and Activity Quantity (Lack of Sufficient Schedule Granularity)
Price Fluctuation Caused Stockpiling	Inconsistent Activity Performance
Shortage Caused Rationing	Resource / Material Constraints

2.6 Conclusions

This research began with the CAPM beta extension of financial portfolio theory and the beginnings of the financial markets, together with the premise that their precepts could be extended to network schedule systems and the expectation of a method to measure the systematic and/or specific risk among its participants. Through the literature, portfolio theory was confirmed as an appropriate means for determining the individual activity specific interactions and risks exhibited by the contingent of activities and their owners, the cadré of subcontractors that comprise a construction project.

Building upon the CAPM beta application demonstrated by exemplar, this research concludes that the specific risk exhibited by schedule system participants can be measured in similar fashion to that of the CAPM beta measure of the risk exhibited by individual assets (stocks, bonds, etc.) to that of the financial market or exchange in which they trade. Returning to Adam Smith's perfectly competitive market, one with no impediments to operation, the as-planned network schedule system can be so ascribed. It is one in which the participants perfectly interact. While financial markets rarely perform in this manner, neither do construction projects perfectly perform to their desired path. Where financial markets developed a measure of this imperfection, network schedule systems (as typically exemplified by the CPM) have not reached this level of sophistication.

By loose definition, a financial beta is the difference between the ongoing performance of an individual asset and a perfectly performing market. It translates to a beta for network schedule systems being the difference between the perfectly competitive schedule and the aggregate performance of individual participants across multiple projects.

Much like the magnitude and sign of a financial beta depict the scale and direction to which an asset correlates to the market, so too can similar correlations and characterizations be made for a schedule system beta. Table 2.19 depicts these assessments.

Table 2.19: Schedule Risk Classification by Beta Range

Beta (β_c) Range	Risk Classification	Characteristic
$\beta_c > 1$	N/A	Not possible – activity schedule duration cannot exceed that of project / project cohort.
$\beta_c = 1$	Extreme Risk	Responsible for ALL schedule delays within the cohort of projects considered
$0.5 < \beta_c < 1$*	High Risk	Responsible for the majority of schedule delays experienced on projects
$0.25 < \beta_c < 0.5$*	Moderate Risk	Performance in excess of that generally expected with typical project risks
$0 < \beta_c < 0.25$*	Low Risk	Performance generally expected within acceptable limits and generally experienced project risks
$\beta_c < 0$	Little Risk (may be considered safe or anti-risk)	Expected to perform better than as-planned schedule durations, despite normally experienced project risks

* The ranges for β_c are not uniformly distributed between zero and one. This is due to the premise and judgment of the author that significance for high risk is better represented at the 0.50 level that at the third-point of 0.67. Likewise, moderate and low risks are viewed as equal components of the lower half of the range (the lower quarter-points).

This research concludes that for a beta approach to schedule risk measurement, performance across multiple projects must be exhibited and correlated. Beta is a measure to market performance, herein a cohort of construction projects, not a singular attribute within a closed schedule system.

2.7 Future Research

This new approach to measuring risk (i.e. the expenditure of float) is based upon previously vetted financial portfolio theory tools and opens extended avenues for future research. This research represents the initial element in the three-part fulfillment of risk quantification, pricing and mitigation, allocation, and the development of a prediction method for where risk is likely to reside and is focused on the identification, measurement, and location component. As a valid risk identification and measurement method has been crafted via CAPM beta, future investigation depicting analogous research and extending the components of the *Total Float Traded as Commodity* notion of de la Garza et al. (1991) is warranted. An as-planned versus as-completed duration database to facilitate the calculation of actual betas for a cohort of network schedule systems and the common activity ownership (subcontractors) should be developed.

It is expected that the remaining elements in this three-part research endeavor will complete the components necessary for a float-trading means will address risk's allocation and value (price and/or cost) within network schedule systems, and in particular within construction project CPM networks. In similar form to the methods employed herein, subsequent research should engage concepts currently vetted, in existence, or in practice. It is expected that a working predictive modeling mechanism will result from the exploration and analysis of risk's residence in network schedule participants along with a market model for its exchange.

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CHAPTER 3

Measures of Decision-Making Power for Quantifying Risk Allocation within Network Project Schedules

Abstract. Risk remains in all construction projects where it resides among the collection of subcontractors and their array of individual activities. It reveals itself in different ways in projects of all sizes and complexities. Wherever risk resides, the interaction of participants to one another becomes paramount in the way that risk is viewed and addressed. Within a network project schedule, inherent risk becomes recognizable, quantifiable, and can be mitigated by consuming float – the flexibility of a project to absorb delays. Allocating, owning, valuing, and expending float has been debated since the inception of the critical path method. This research investigates the penultimate element of a three-part treatise that examines how float can be traded as a commodity, an unrequited concept in construction engineering and management whose promise remains unfulfilled for lack of a holistic approach. European voting models and the ability to change the outcome of an election are explored as analogies for the interactions of participants in construction projects. Disparate entities, herein subcontractors, participate in a complex decision-making process subject to constraints and uncertainties regarding project timeliness. Risk measurement is explored through the Penrose square root law, which allocates voting power based on the square root of the population, to determine parallels to the risks represented by each schedule activity. This research represents the theoretical extension required for the allocation of the risk mitigation means to develop a trading model across a network schedule system's critical participants.

3.1 Introduction

Risk is ever-present in construction projects. Although much has been written about its cause, effect, mitigation, avoidance, and its transfer or shifting through numerous techniques, programs, contractual terms and conditions, or insurance vehicles, there remains no standard practice for its valuation or allocation. Risk, beyond its physical appearance through weather conditions, project-related mishaps, or other external influences (project changes, building code changes, non-conforming work, etc.), is most recognizable and quantifiable by its impact on a project's schedule, as measured by consumption of its float.

This research, the second in a three-part approach to modeling how systematic risk can be quantified, priced, diversified and/or mitigated, and the development of a prediction method for where risk is likely to reside, is focused on the allocation of risk among a construction project's participants. It is an extension of *Total Float Traded as Commodity* (de la Garza et al. 1991) and is rooted in social decision-making models.

3.2 Literature Review

Risk in general is simply defined as the effect of uncertainty on objectives, whether positive or negative (ISO 2009), and more specifically as the probability or threat of a damage, injury, liability, loss, or other negative occurrence caused by external or internal vulnerabilities, and that may be neutralized through pre-mediated action (Webfinance 2010a).

3.2.1 Risk Categorization

Risk is represented by, can be found in, or can arise from four basic categories (Artto and Kähkönen): pure risk (general hazards and weather conditions); financial risks (cash flow), credit risks, or bankruptcy; business risk (almost anything that may occur during a project); and political risk (the extreme conditions surrounding the political environment such as changes in governments, laws or regulations by fiat, with the most extreme being international and/or cross-cultural hostilities).

Table 3.1: Project Risk Categorization – External to the Schedule Network (Most Broad to Most Defined)

Miller and Lessard (2001) (broad)	**Turner (1999)**	**Artto and Kähkönen (2000)**	**Finnerty (1996)**	**Miller and Lessard (2001) (narrow)**	**Baloi and Price (2003)**
Institutional	External	Political / Country	Political	Sovereign	Political
				Social Acceptability	Social
				Regulatory	
		Pure	Environmental		Natural
			Force Majeure		
Market	Insurable	Financial	Financial	Financial	Financial
			Supply	Supply	
			Economic		Economic
				Demand	
					Commercial
			Currency		
Completion	Business	Business	Technological	Technical	Technical
			Completion	Construction	Construction
	Internal			Operational	Logistics

Further studies (Baloi and Price 2003) classify risk specific to construction projects via an eightfold categorization: technical, construction, legal, natural, logistics, social, economic, financial, commercial, and political. Similarly, risk can be viewed from differing perspectives, from where risk resides (Turner 1999), risk's source (Miller and Lessard 2001), from a financial perspective (Finnerty 1996), or relegating risk to those factors beyond the control of the project's participants, namely external risk factors (Hallikas, Virolainen and Touminen 2002). They are summarized and aligned in Table 3.1. On the other hand, Baloi and Price also categorize construction risk from its pairwise impact classification: dynamic vs. static, corporate vs. individual, internal vs. external, positive vs. negative, acceptable vs. unacceptable, and insurable vs. non-insurable.

Risk, and in-particular risk related to construction activities and its manifestation within network schedule systems, can be mitigated. Four primary strategies exist to accomplish this: transference – shifting the risk to another party, avoidance – forestalling the risk, mitigation – reducing the negative effect of the risk, and/or acceptance – realizing some (or all) of the consequences of the risk.

3.2.2 Risk Management and Classification

Risk management, the manifestation of activities to overcome risk (inherent or systematic) is defined as the process of determining the maximum acceptable level of overall risk to and from a proposed activity; then using risk assessment techniques to determine the initial level of risk; and, if it is excessive, developing a strategy to ameliorate individual risks until the overall level of risk is reduced to an acceptable level (Webfinance 2010b). It has been formally defined from a project management perspective by the Project Management

Institute (PMBoK 2000) as, "…the process of developing options and determining actions to enhance opportunities and reduce threats to project objectives." As it applies to this research, risk management is defined as controlling the probability, and/or severity, of potential adverse events so that the consequences are within acceptable limits.

3.2.3 Translation to Network Schedule Systems

The unifying factor among all risk classifications is the potential to impact a construction project's schedule; herein identified as schedule risk: the need for a project (schedule) to exhibit flexibility to absorb externally-influenced and/or externally-originated delays. It is inextricably linked to and defined as a project's float, which can be impacted in two ways: positively – where the time required to complete individual tasks is less than the schedule identifies, causing the schedule to advance or finish quicker, or negatively – where task durations are exceeded, and schedule delays can be expected.

Unfolding from risk identification and classification is the source or cause of construction project risks and uncertainties: the cause of negative impacts to construction project schedules. Seven primary factors causing significant delays on construction projects are owner interference, inadequate contractor experience, financing and slow payments, labor productivity, slow decision-making, improper planning, and subcontractors (Odeh and Battaineh 2002). All are risks that can be viewed through network scheduling.

3.2.4 Decision-Making Models

The way in which individuals, groups, organizations, and political entities combine their preferences, needs, and choices into a decision or overcome a perceived risk is a question about which there has been considerable speculation, research and mathematical modeling. In democratic capitalist societies, there are essentially two primary methods to make choices or decisions (Arrow 1964): voting (typically used to make political decisions); and the mechanisms of the marketplace (typically used to make economic decisions). Groups may engage in majority votes with or without veto power, be subject to dictatorial leadership models inclusive of unilateral decisions by fiat, form collective bargaining agreements, evaluate options through mathematical models, decide by governing board and consensus, or they may employ economic value-based methods. Group decisions or social choices are made in many ways with mechanisms unique to the specifics of the group. Larger and more complex groups tend towards voting and elections, where smaller groups favor committee structures and consensus (Lieberman 1971).

Irrespective of specific decision-making approach, the issue of combining individual preferences into socially acceptable outcomes or choices has both normative and descriptive aspects. The normative aspect of the social choice question is described in its basic form as: how *should* groups combine individual preferences into a sensible consistent decision to produce results. The descriptive question is how *do* individuals and groups meld their preferences into an agreeable decision. The commonality between these diverse aspects of social choice is conflict resolution resulting from contradictory preferences, traditions or customs. The voting method and those left to the marketplace represent the amalgamation of individual tastes and preferences into the decision-making process, yet both involve a collective choice among a limited range of alternatives or opportunities.

Allocating, owning, valuing, and expending float in network schedules has been debated since the inception of the critical path method and is the contradictory preference at hand. The choice to expend float is the ultimate conclusion or decision reached in the risk management progression in network schedule-based construction projects. The various determining processes the members of the group or organization (the collection of subcontractors and the general contractor as applies to this research) undertake has six general elements (Lieberman 1971):

1. The distribution of power
2. The joint welfare function
3. Bargaining and coalition processes
4. Individual differences and characteristics of the participants
5. Group processes or phenomena
6. Previous experiences and commitments of the group members, and the possibility of future interaction

The preponderance of these elements can be found in the decision to expend float, with the distribution of power, or the lack thereof, being of foremost concern. One of the most common processes by which social choice is made, the manner in which the collective integrates individual preference into a decision, is by voting (Birnberg and Pondy 1971). Within the distribution of power through voting, three characteristics emerge as the determining elements upon which rests the satisfaction of the participants: size – the number of persons and of issues; distribution – how preferences on an issue are distributed over individuals; correlation – the correlation, over issues, between the preferences of one entity or person and that of another or others.

3.2.5 Voting Practices

Decision rules and the exercise of control through voting power is very complex. Some frameworks for voting power consider the aforementioned determining elements: size, distribution, and correlation. The bicameral form of the U.S. Congress – a representative democracy, and the European Union (EU) – a federation of independent countries, are among the most contemporary and comprehensive.

Regrettably, neither the representation nor decision-making embodied by the U.S. democratic republic or the EU federation is *directly* analogous to network scheduling, given the indirect nature of the decision-making rules. The direct representation envisioned by the Athenians and their *dēmokratía*, the "rule of the people," during the 5th – 4th century BC (Ober 2008) is not embodied in contemporary practice. In the U.S., the election of a president and the activities of Congress (passing of legislation (bills and resolutions), treaty ratification, budget enactment, or the confirmation of senior non-elected officials), are made by those elected by popular vote to represent the electorate, not directly by individual constituents. The primary characteristic of a representative democracy is that while representatives are elected by the people to make decisions and act on their behalf, they retain the freedom to exercise their own judgment in doing so. This gap between the constituency and the vote for a decision rules out a direct correlation to the inner workings and decision modes found in network schedules.

Likewise, the political system of the European Union, a *federation* of 27 nations (EU 2010) of different population size and economic power (several of which have their own internal federal systems) that share common interests and heritage, fails to overcome the analogous shortfalls of the U.S. republic. Where it differs and becomes *apropos* to this research is that mathematicians have examined the EU's voting process at the supranational level (Scientists for a Democratic Europe 2004) to validate its tenets or search for appropriate alternatives to its representative voting approach.

In such a process, all representatives of one country cast the same vote on behalf of their people, either for or against a decision (a bill), thus leading to a multi-level system: the population, their elected representatives, and the final decision of a vote by said representatives. It was discovered that widely held intuition will lead to an erroneous understanding of its decision-making (Kirsch 2004). Such intuition on voting weight would either demand a "one vote per person" or a "one vote per country" approach (Słomczyński and Życzkowski 2007) to reach a majority and thus determine the outcome of a vote. However, neither of these extreme cases within the spectrum of possible federal-type electoral systems is fair and equitable from a scientific perspective. The former would favor large countries so that they could always overpower smaller ones, which would establish a *"tyranny of the majority"* (Mill 1913, p.3, emphasis added). Conversely, the latter would favor small countries so that they could always block larger ones, which would contradict the principle of majority rule.

This conundrum led to negotiations for the new (actual) EU solution to contain elements of both approaches in a double majority system (Hosli 1995). Specifically, per the

Treaty of Lisbon (EU 2007), which becomes effective in 2014, there will be the requirement for a minimum of 55% of the votes in the Council of Ministers and representation of at least 65% of the population for a bill to be accepted. The current Treaty of Nice (EU 2001) more stringently requires a triple majority by number of countries, ministerial votes, and population.

In specific practice, the EU Council of Ministers, together with the European Parliament, comprises the bicameral EU legislature (Congleton 2003) and is composed of one minister per country, who casts multiple identical votes (a bloc) in a weighted voting process. The number of a minister's votes is fixed at even integer value that appears to be clustered for large, medium, and small population sizes. Mathematically, such a decision-making process is assessed by examining the theoretical voting power of one voter or bloc (i.e. a minister).

3.2.6 Analytical Voting Models

Two voting models provide insight into the allocation of float, the *a priori* measure of risk within network-based schedules. Seminal work by Penrose (1946, p. 53) provided an insightful approach to voting power that was later repurposed by Banzhaf (1965):

> In general, the power of the individual vote[r] can be measured by the amount by which his chance of being on the winning side exceeds one half. The power, thus defined, is the same as half the likelihood of a situation in which an individual vote can be decisive – that is to say, a situation in which the remaining votes are equally divided upon the issue at stake. The general formula for the probability of equal division of *n* random votes, where *n* is an even number, approaches $\sqrt{2/n\pi}$ when *n* is large. It follows that the power of the individual vote[r] is inversely proportional to the square root of the number of people in the committee.

The essential principle is that the number of voters per bloc itself does *not* matter directly. Rather, a federal system uses a *multi-level election*, i.e. voters elect representatives (EU ministers), who in turn cast a bloc vote for the country, which together make a decision (e.g. decide for or against a bill). At each level, the majority rule creates a winner-takes-all outcome. Thus the actual voting power of a particular bloc is directly proportional to the *percentage of permutations* (among all possible blocs on either side of a decision) where the decision of only that bloc will sway the entire election (Gelman et al. 2002).

3.2.6.1 The Work of Lionel Penrose

The work of British psychiatrist and mathematician Professor Lionel Penrose is recognized as the earliest scientific work on the measurement of voting power. In his short paper, Penrose proposed a probabilistic measurement of hypothetical votes in the newly formed United Nations General Assembly. The seminal paper argues that the equitable distribution of voting power in the assembly should be proportional to the square root of the population represented or served. He was concerned that,

> If a committee or electorate consists of two sections, a 'resolute' block and an 'indifferent' random voting group, a small 'resolute' group, who always vote together can exercise a surprisingly powerful control over the whole committee. Thus, three resolute votes can control a committee of twenty-three could control, again to the same extent, an electorate of over 1,000… (Penrose 1946)

Penrose's idyllic voting scenario presented the case in which every responsible human being should have equal power in a world assembly (the UN General Assembly *per se*), but he recognized that this would only be possibly wherein the assembly would be formed of nations of equal size. Absent this, he concluded that, "…if large and small nations have equal voting powers, the spokesman for small nations are felt to be too significant, and artificial rules about the meanings of votes and vetoes have to be constructed to redress the balance” (Penrose 1946).

In his search to overcome this and for an optimal two-tier voting system (where a set of constituencies of various size elect one delegate each to a decision-making body) in which every citizen of every country has the same potential voting power, Penrose first considered a direct election in a state consisting of *n* voters and proved that the voting power of a single citizen decays as $1/\sqrt{n}$, provided that the votes are uncorrelated. To compensate for this effect, he suggested that the *a priori* voting power of each representative in the voting body should behave proportionally to $\sqrt{n}$, making the citizens’ voting power in all states equal and so the whole system is representative, and the two factors cancel each other out $\left(\frac{1}{\sqrt{n}} \bullet \sqrt{n} = 1\right)$. This concept became eponymously known as the Penrose Square Root Law.

Table 3.2: Determination of Resolute Block Decision Control over Indifferent Populations

Indifferent Population (*n*)	**Percentages of Decisions Controlled by Resolute Blocks**		
	84.1%	**97.7%**	**99.9%**
	Population of Resolute Block		
25	5	10	15
100	10	20	30
10,000	100	200	300
1,000,000	1,000	2,000	3,000
100,000,000	10,000	20,000	30,000
2,500,000,000	50,000	100,000	150,000

n voters with a resolute block size of $\sqrt[x]{n}$ can carry $\frac{1}{2}(1+a)$ decisions, where *a* is the area under the normal probability curve (Penrose 1946)

The important statistical fact that emerges from Penrose's work is the high degree of control exercised by a relatively small group when the indifferent population served is high, as is depicted in Table 3.2.

3.2.6.2 The Square Root Law

The conceptual reason for assigning voting weight in proportion to the square root $\left(\sqrt{n} = n^{0.5}\right)$ of the population *n* of each country (the other factors in the formula being constant) is that this strikes an ideal balance exactly between the aforementioned *one vote per country* that is proportional to n^0 versus *one vote per person* that is proportional to n^1 (Kirsch 2007a, Pöppe 2007). This assumes a borderline scenario where all other blocs are equally divided into two coalitions, for and against the decision, both remaining in the minority. In the original derivation of the square root law such coalitions were assumed to be independent and equally likely, which omitted any realistic influence by political, economic, or even cultural factors (Słomczyński and Życzkowski 2007). However, this can be remedied by moving the exponent of the formula more toward either of the two extremes; a value above 0.5 would recognize stronger correlations within each country, i.e. a collective bias that practically amounts to national interests and values (Kirsch 2007b, p.359, Rieck 2007).

3.2.6.3 The Work of John Banzhaf

The second landmark work in voting power is named after John F. Banzhaf III, a U.S. electrical engineer and attorney, and is better known than its antecedent from Lloyd S. Shapley and Martin Shubik. The Banzhaf Power Index addresses voting power from a legal-constitutional perspective. It came about in an unsolicited response to a 1960s lawsuit involving Nassau County, New York and the practice of state legislatures and county boards attempting to meet the requirements that in representative assemblies equal numbers of citizens have substantially equal representation. Boards like Nassau attempted to fulfill this requirement by using weighted voting and assigning to the board members weights proportional to their respective constituencies.

Where Penrose sought to rebalance the voting power of diverse blocks, Banzhaf's work focuses on the relative power of the "deciding, critical, or swing" vote, the one that specifically changes the outcome of an election. By his definition, voting power is derived from the ability to change the outcome of a decision, not from the relative ability of a larger group to *de facto* control the outcome. Banzhaf defines "critical" as anyone who holds enough votes to change the coalition from a winner to a loser, or vice-versa (Banzhaf 1965). It may be a single vote as proposed by the earlier work of Shapley and Shubik (1954), or a collection of "enough" votes, a bloc of votes, as Banzhaf posits.

3.2.6.4 The Voting Power Index

Banzhaf developed an index representing the probability of a single vote or constituency changing the outcome of a vote where voting rights are not necessarily equally divided. The index, the power of an individual voter to change the outcome of a vote, is calculated by first identifying all of the vote "winning" coalitions and then counting the critical voters – where a critical voter is one who, if he changed his vote to that opposite from that of the winning coalition, would cause the vote to fail. The index is measured by the fraction of all swing votes that the voter could cast. The Banzhaf Power Index for Player (Voter) 'P': BPI (P) is:

$$\text{BPI (P)} = \frac{\text{\# of times Player (Voter) `P' is critical}}{\text{Total \# of times all Players (Voters) together are critical}} \qquad \text{[Eq. 3.1]}$$

The BPI is based upon the assumption that players can freely enter or leave the voting coalition and that the player's power is proportional to the number of times he is critical, i.e. the number of times the player or voter is the deciding vote that changes the outcome. It is computed by employing the following process:

Step 1: Determine all winning coalitions.
Step 2: Determine the critical players in each winning coalition.
Step 3: Find the number of times all players are critical.
Step 4: Find the number of times Player P is critical.
Step 5: *BPI (P)* is the smaller number (from Step 4) divided by the larger number (from Step 3).

3.2.7 Integer Rounding – Sainte Laguë and d'Hondt

Several methodologies exist for the proportional allocation of seats within representative assemblies (Pukelsheim 2007): the Sainte-Laguë method (found to be equivalent to Webster's method) and the d'Hondt method (mathematically equivalent to Jefferson's and the Bader-Ofer methods, but different operationally). Irrespective of their specific formula for proportional allocation (not a consideration herein), a common element is found that applies to this research: votes are allocated in integer values. Partial values or fractional voters are an impossibility, as is a fractional day for float allocation and/or consumption. Hence, float values used herein will be rounded to the nearest integer and adjusted in their aggregate such that their sum matches the total float calculation.

3.2.8 Game Theory

Mathematical models serve as vehicles for the study and prediction of varying types of systems (the aforementioned voting models being a subset thereof). The development of such models came into prominence first through the applied sciences and then over the past half-century has found roots in the behavioral sciences, in particular that of social choice and decision-making (Shubik 1964). Within the sciences, mathematical models are able to characterize and synthesize with sufficient accuracy the relationships among the various factors of the system under study (e.g. population, physical properties and/or attributes, etc.). The elements or situations of interest in the behavioral sciences, those akin to voting, are subtle and more complex. They surround the interactions among participants, which may or may not be consistent or logical, and can be entirely unclear and even poorly formulated.

The application of these models, the study of uncertain decision-making in situations of competition and conflict and/or cooperation and interdependence to determine an optimal course of action, is known as *game theory*, with the most prominent behavioral models or "games" being the Prisoner's Dilemma and the Tragedy of the Commons. Game theory is focused on human processes in which the individual decision-making entities are not in complete control of their respective outcomes. The kernel of it is that the fates, objectives and/or results of the participants are intertwined and mutually dependent, with many possible outcomes with differing values. These outcomes are almost always nonconstant sum games (Shubik 1964). That is, the individual participants stand to gain by joint action, in spite of having differing and opposed interests; one entity need not lose for the other to gain. For example, synergies are developed in the game between labor and management that are mutually beneficial and compound as the game proceeds, whereas destruction occurs with war games. Therefore, the mathematics of the zero-sum game do not apply to the behavioral science component of game theory.

3.3 Analogy to Construction Network Schedules

3.3.1 Correlation of Voting Measures to Float

Disparate entities, herein subcontractors, participate in a complex decision-making process subject to constraints and uncertainties as to whether the project will be completed on time. Where votes comprise an election, many risks and uncertainties compose a project. Building upon social choice and decision-making, voting theory typify a possible determinant for the analogous behavior to those participating in large construction projects; and a potential element of a risk management or mitigation approach to network schedule uncertainties and their negative impacts.

Despite the apparent large difference between political voting and project management practices, numerous conceptual analogies can indeed be identified. Table 3.3 provides a detailed comparison of their elements at varying levels of detail. A federal system, whose elected representatives campaign and vote toward a decision (for or against a bill), is analogous to a schedule network, wherein non-self-performed (i.e. outsourced) construction work is planned and performed by different subcontractors. Both elections and projects are modeled as binary decision-making processes. Voters have a quantifiable influence (their voting power) on the decision, so do subcontractors have some (as of yet unknown) potential influence on whether the project will be on schedule and within budget.

Table 3.3: Conceptual Comparison of Decision-Making in Political Voting and Project Management

Voting Elements	**Project Management Correlation**
Federal System	Schedule network
Representation	Non-self-performing, outsourcing
European Union	Project participants
Council of Ministers	All subcontractors
Election (Three Levels)	Project (Three Levels)
Campaigning	Planning
1. Weight *(Population)*	1. Value or duration of activity by a single subcontractor
2. Voters *(Ministers for their countries)*	2. Subcontractors *(Different specialties of crafts and trades)*
3. Vote *(Yes/No, For/Against)*	3. Decision to Expend Float *(Yes–Expend/No–Accelerate)*
Ballot (Voting process)	Schedule and budget (Project controls)
Undecided	Available Float (different types)
Polls	Updates
Counting	Outcome assessment
Majority	Impact on schedule or budget
Winning	On time, within budget
Losing	Delayed, over budget
Voting Power	Influence on Performance
Swing vote	Critical participant

The population size, i.e. voting weight, of the different countries is analogous to the duration or monetary value of the different activities that subcontractors perform. Another quantitative measure of the potential weight of each subcontractor is the risk factor from accident insurance ratings for the specific type of work and individual company (not addressed herein). Additional analogies exist between voter polling to forecast the decision and updating the schedule and budget to forecast the project performance. Counting votes or performing an outcome assessment, respectively, yields the results of these analogous decision-making processes.

Furthering the analogy to voting, Gelman posits that the next step in evaluating voting models, patterns and power is to "…give a dependence structure to the voters' probabilities… [where] it makes sense to build this dependence upon existing relationships among the voters" (Gelman et al. 2002), a reference to the geographic structure in the U.S. of states, Congressional districts, counties, cities, precincts, etc., or conversely, the 'softer' demographic specific to the multiplicity of constituencies forming the electorate. Herein such a dependence structure based upon on existing relationships remains with the collection of subcontractors and the risks and intricacies nested within network schedules, in particular, float.

Float is a measure of flexibility that reduces risk and increases opportunity. It quantifies the ability of an entire schedule or individual activities therein to absorb delays. As a vote is the vehicle by which decisions are made, the decision to expend float is the analogous vote within network schedules. The only significant conceptual difference between elections and projects is that activities in a construction project occur within a dependency structure per the schedule network, whereas blocs cast their votes all at the same time into the ballot box. This initially creates a challenge for investigating how these voting models can be converted into the basis for allocating float within network schedules in the construction industry, and secondly which (or possibly all) of the various measures that characterize subcontractors and their activities would be most suitable for inclusion in such an allocation model.

3.3.2 The Critical Path Method

Traditional project planning is strongly centered on the discrete scheduling of time. Scheduling as currently practiced originated in 1956 when Kelley and Walker (1989, 1959) planned a factory with an early UNIVAC computer. Their *critical path method* (CPM) adds deterministic durations from the overall start forward along *paths of fixed dependency* and compares these *early dates* with subtracting durations backward from the overall finish for *late dates.* Links connect starts (S) and finishes (F) as either F-S, S-S, F-F, or S-F and may carry lags. It is not widely recognized that this recursive algorithm is a *simplified case of linear programming* (Dantzig 1955) used to solve a system of equations (i.e. start + duration = finish).

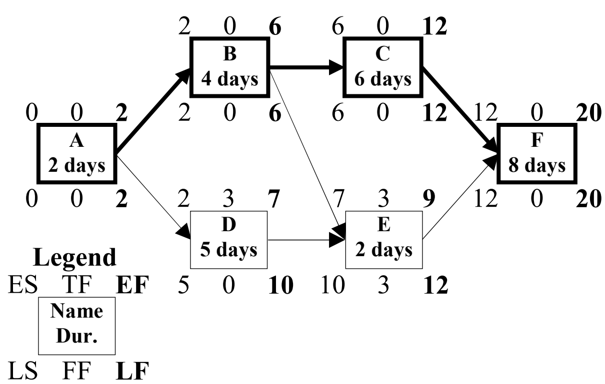
Critical path method (CPM) calculations are considered ubiquitous (Galloway, 2006) to today's construction projects and are the preferred management tool of contemporary contractors and project managers. CPM facilitates forecasting to an accepted level of accuracy the allocation of time to complete the construction project, as well as the representation and inter-relationship of individual tasks into which the overall project may be divided. A CPM schedule is designed to advise the parties involved about the relative importance of performing certain activities within the project completion parameters within the predetermined duration and sequence.

3.3.2.1 Description of the Critical Path

When activities are knitted together to form an overall schedule, variations in activity durations compensate each other, resulting in a reasonably accurate network schedule system. Upon determining the applicable classification for an activity (predecessor, successor, or concurrent / independent), and the forward and backward passes (the early start and early finish times and late finish and late start respectively) the critical path (the longest uninterrupted path from beginning to end of the project,) can be determined and the critical activities along the critical path become apparent.

When the duration of an activity is not equal to the late times computed by way of the forward and backward passes, float arises. That is, there is additional time in which the activity can be completed before impacting the successor activity's start, for which there are multiple classifications, including free float, total float, independent float, interfering float, etc. Figure 3.1 shows exemplar activities with durations below, plus early and late start and finish dates, and float as calculated by CPM. Differences of forward and backward dates yield float, i.e. the flexibility of activities to absorb delays. Zero float is considered critical, hence the descriptive naming of the continuous path(s) through the schedule network.

Figure 3.1: Simple CPM Network Schedule



Total float is a byproduct of CPM calculations and represents the length of time an activity's finish date may be delayed without affecting the completion date of the entire project (de la Garza et al. 1991). Total float is an attribute of the network path and is not assigned to any one specific activity. Its consumption will reduce the total float times for subsequent activities. It is shared with the other activities along the same path (Callahan et al. 1992), and cannot be owned by individual activities on the path.

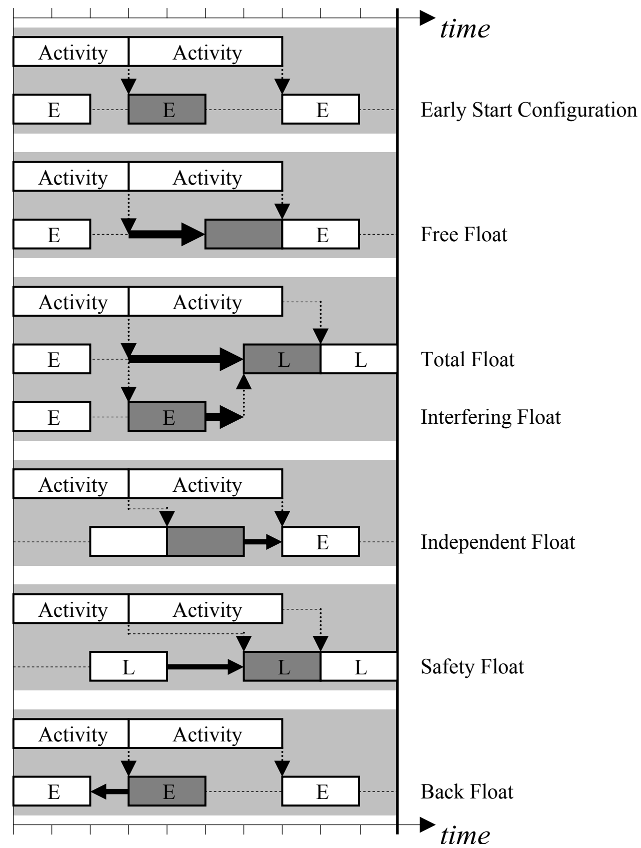
Both owners and contractors embrace the flexibility of activities that have positive float associated with their durations. It is used to deal with unanticipated conditions or uncertainties (risks) arising over time or to accommodate changes in the work. Raz and Marshall (1996) view total float as the "important degree of scheduling flexibility" associated with the activities in a network schedule system.

3.3.2.2 Float Definitions

Float is measured in multiple ways and defined differently depending upon the viewpoint, need, or relative position within a network-based schedule. Among the more common and well know float models are the following, which are graphically depicted in Figure 3.2.

Float: Lag time, slack, or buffer (negative float also called 'lead') that the start of an activity can be delayed within the schedule; lag and lead often refer to mandatory buffer durations between two activities, e.g. to allow concrete to cure

Figure 3.2: Graphical Representation of Float Types from Critical Path Method (CPM)



Total Float: Possible delay of activity *i* without impacting project end date, is therefore of interest for project owner, shared by all activities along one path, critical path has zero float, also: task float or path float

$TF_i = LS_i - ES_i = LF_i - EF_i$

Start ~ Total float of activity i calculated using only start dates

$Start\ F_i = LS_i - ES_i$

Finish ~ Total float of activity i calculated using only finish dates

$Finish\ F_i = LF_i - EF_i$

Free Float: Possible delay of activity *i* without impacting any of its successors *j*, is therefore of interest for project subcontractors ,not shared but unique to the individual activity, also: real float or internal float or activity float. Summing FF along <u>*one*</u> path gives the TF.

$FF_i = (min\ ES_j) - EF_i$

Interfering Float: Difference between total float and free float of activity *i*, the part of the total float that affects only other activities but not the project end

$Interf\ F_i = TF_i - FF_i$

(Halpin and Woodhead 1998, Elmaghraby 1995 called interference float, Leu et al. 2000 called interfacing)

Independent Float: Possible delay of activity *i* if all predecessors *h* finish as late as possible and all successors *j* start as early as possible, thus squeezing from both ends (Halpin and Woodhead 1998)

$Indep\ F_i = (min\ ES_j) - DUR_i - (max\ LF_h)$

Safety Float: Possible delay of activity *i* if all predecessors *h* finish as late as possible (Thomas 1969, Elmaghraby 1995)

$SF_i = LS_i - (max\ LF_h)$

Back Float: Possible time span in which activity *i* can be started without forcing any predecessors *h* to be finished earlier than their earliest finish

$BF_i = ES_i - (max\ EF_h)$ (Harris 1978)

$BF_i = min\ (ES_i - EF_h)$

$BF_i = min\ (LAG_{i\ h})$

Current Float: Uses current time (CT), can be negative if activity is behind schedule, "defined as the finish float available to the activity with respect to the latest finish time of that activity"

$CF_i = LF_i - CT - DUR_i$ (Shanmuganayagam 1988, p.403)

3.3.3 Float Ownership and Allocation Approaches

Issues and constraints that influence float allocation have been identified in previous research (Al-Gahtani 2009). Multiple definitive approaches to the ownership, allocation, control of and expenditure of float can be found in practice to compensate for project risks and schedule overruns. They fit within three main categories: owner entitlement, contractor entitlement, or project entitlement.

Owner Ownership of Float: The position that due to the owner paying the costs of a project, he is entitled to ownership of float. In practice, owners manage float to lower their project-associated risks that are generally assigned to them via the contract (Pasiphol and Popescu 1995; Prateapusanond 2004).

Contractor Ownership of Float: Sole ownership of float by the contractor due to his assumption of the means and methods responsibility for construction and his control over the activities and sequence of construction; the implied right to manage the project schedule (in total), workforce, equipment, cash flow to deliver a project on time and on budget.

Project Ownership of Float: The most common approach found in case law that gives the owner and contractor the right to consume float. It is the simplest method, as it makes float available to all parties based upon the first use premise. This has the potential to increase a contractor's risk absent compensation (Arditi and Pattanakitchamroon 2006).

Emanating from the ownership categories are approaches to the management of float among primary entities.

Equal Proportion Approach: A recent introduction that integrates all parties and established a common understanding that total float is initially allocated equally between the owner and contractor. As the project ensues, float consumption is tracked. When a party consumes more than his 50% allocation and a noncritical path becomes critical, the delay to the critical path is equally shared between the owner and contractor based on their respective ratio of disentitled float consumption. For paths that are not critical, none of the parties is entitled to float consumption, as it will be governed by the project float approach. Its intent is to formulate a rule for consuming the float that affects the critical path delay such that the parties remain cautious when consuming the float (Prateapusanond 2004).

Bar Approach: This approach accounts for critical and noncritical path delays by considering each specific consumption of float as a critical delay, for which each total float activity of the corresponding activity is represented as a bar in a bar chart. While the project approach to float poses no restrictions on float consumption, the bar approach greatly limits its use to minimize the overall effect of its consumption and thereby prevents disentitled float consumption by both the owner and the contractor (Ponce de Leon 1986).

Contract Risk Approach: This approach establishes a link between float ownership/consumption and the assumption of project risk. Contractors own the float if they assume full responsibility for the project risks, as is done in a lump-sum contract; owners own the float if they have full responsibility for the project risks, as is done in cost-plus-fixed-fee and cost-plus-percentage-fee contracts. When both the owner and the contractor share the project risks, such as cost-plus-fixed-fee contract with a maximum or percentage ceiling, the owner and the contractor agree to a ratio to share the float proportional to their respective risk levels (Householder and Rutland 1990).

Commodity Approach: This 'time is money' approach treats float as a commodity that is tradable between the contractor and the owner while considering the effects of float consumption on project risk. It gives the contractor complete control over float, as he assume the risk in a lump-sum contract, and allows the owner to consume float (if needed) by purchasing it from the contractor based on an agreed to contractual formula (de la Garza et al. 1991):

$$\begin{array}{c}\text{Daily Trade-in Total Float (TF)}\\ \text{Value for Each Activity}\end{array} = \frac{\text{Late Finish Cost} - \text{Early Finish Cost}}{\text{Total Float (TF)}} \qquad \text{[Eq. 3.2]}$$

Path Distribution Approach: This approach is that total float for each noncritical path can be distributed based on the activity duration. It starts from the nearest critical path (second longest path after the critical path) and proceeds to the next-nearest critical path (third longest path after the critical path) until completion of all the noncritical paths. In each path calculation, total float is distributed proportionately to the noncritical activities based on their durations. Before moving to the next nearest critical path, distributed total float for all noncritical activities on the path being analyzed should be adjusted such that its activities' durations become the critical path (Pasiphol and Popescu 1995).

Day-by-Day Approach: A formula approach focused on solving the associated issues of changing total float as a result of impacting the critical path tracks float consumption of the owner and contractor, which can be used to determine the respective disentitled consumption. This approach states that any of the parties who change float as a result of delaying the critical or noncritical paths will get credits or debits for impacting the critical and noncritical paths based on a day-by-day, systematic assessment and is integrated with a delay/acceleration project analysis. At the end of the analysis, all contract parties have an account (credit / debit) for recording their entitlement float consumption, their project delays, and their actions that accelerated the project schedule (Al-Gahtani and Mohan 2007, Al-Gahtani 2006).

Total Risk Approach: Risk-based float ownership is based on the principle that the party who carries the greatest project risk should be entitled to the greatest consumption of float and deserves compensation from other project parties who consume more than their proportionate share of the available float. In a lump-sum contract, contractors should own float that is equivalent to their risk, and those who use float beyond their allocations should be liable. For a cost-plus-fixed-fee contract, the owner (rather than the contractor) should own float that is equivalent to his risk, and those who use float beyond their allocations should be liable. The contract risk between the contractor and subcontractor or between the owner and other contractors can be integrated with the activity duration for sharing this contract risk according to the following integrated formula (Al-Gahtani 2009).

$$\begin{matrix}\text{Total Float (TF)}\\\text{Distribution}\end{matrix} = \frac{\text{Noncritical Activity Duration}}{\begin{matrix}\sum \text{of Noncritical Activities Durations}\\\text{on the Nearest Critical Path}\end{matrix}} \cdot \text{Contract Risk Factor} \qquad \text{[Eq. 3.3]}$$

Where the *contract risk factor* ranges between 0 and 1 and is assigned in proportion to a party's risk liability. This factor is related to risk distribution that is written into the project contract and to the types of risks that are related to project conditions, project delivery systems, and kinds of activities necessary to complete the project.

Irrespective of individual approach to float, the terms of the contract allocate the risk among participants (Prateapusanond 2004). These risks should govern the allocation and exchange of float among the project participants. The ownership of such float should be decided in advance and allocated among the parties based on their comparative risk levels (Al-Gahtani 2009). Such an approach decreases the likelihood of project disputes related to the use of float. However, as contracts are rooted in negotiations, not in scientific-based approaches, rigorous mathematical models or analysis, supplemental methodologies are necessary to address the schedule-based total float issues of ownership, allocation and consumption, and the associated risks.

3.3.4 Legal Perspective on the Ownership of Float

Total float is recognized as a by-product of CPM calculations and may or may not exist if activities have not yet commenced or if they were not started on the date initially set forth in the baseline schedule. It is also an expiring time unit if not consumed when available and/or assigned to a particular activity. That is, it disappears if not used as time progresses and work ensues, or delays are incurred that require consumption of float to keep the project within the bounds originally planned.

Irrespective of the cause of the delay or the specific entity consuming float, case law supports the principle that float is available to all entities involved in the work and is not the specific property of any one party. It is "an expiring resource available to all parties involved in a project" (Wickwire, et al. 1991). Public procurement contracts typically include language prohibiting designation of float for the specific and/or exclusive benefit of any one party involved in the work. These clauses are also predicated upon the premise that the first entity to use the available float or a portion thereof is entitled to sole gain the benefits.

Building upon this, Wickwire et al. present the following legal perspective of float:

- Time cannot be stopped, saved or stored.
- Activity durations change as does schedule logic as work progresses that may increase or decrease available float.
- Inclusion of actual start and completion times likewise may increase or decrease available float.
- The inclusion of fragnets (additional changed schedule activities), in a network schedule system representing changes in the work (change orders), delays incurred, etc. can change the position of float in a schedule.
- Contractually extending milestone dates and/or project completion (time extensions) may increase available float.

Given this, entities with some available float time are free to use it for their specific benefit without regard to the impact it might have on other participants in the schedule system or to the overall project completion time.

3.4 Float / Risk Allocation

Drawing from Penrose, the square root law represents a unique concept for the allocation of float (and risk, as previously correlated to float) in network construction schedules. To translate this concept to network schedule systems and determine whether there is a better methodology for allocating float among the participants, this research presents calculations, analysis, and conclusions by way of an exemplar.

3.4.1 Research Expectation

The overarching expectation is that this research depicts a method for the measurement, allocation and pricing of risk within network schedule systems as represented by the consumption of float that addresses the unique treatment and understanding of total float. Float is a vanishing commodity that it is generally consumed on a first-come, first-served basis and is not owned by any single entity (owner or contractor) or participant (subcontractors). More importantly this segment of the research triplet seeks a method that defines an equitable means for allocating tradable for exchange among the participants most in need of its flexibility: critical network participants (those residing on the critical path that by definition have no available float).

3.4.2 Exemplar Development

Reviewing the body of literature and analogous research, a simple network schedule used to depict network complexity and differing time calculation methods (Lucko 2005) is expanded to depict a project whose attributes and performance can easily, but accurately, portray the concepts under development and lend credibility to its analysis, conclusion(s), and extension.

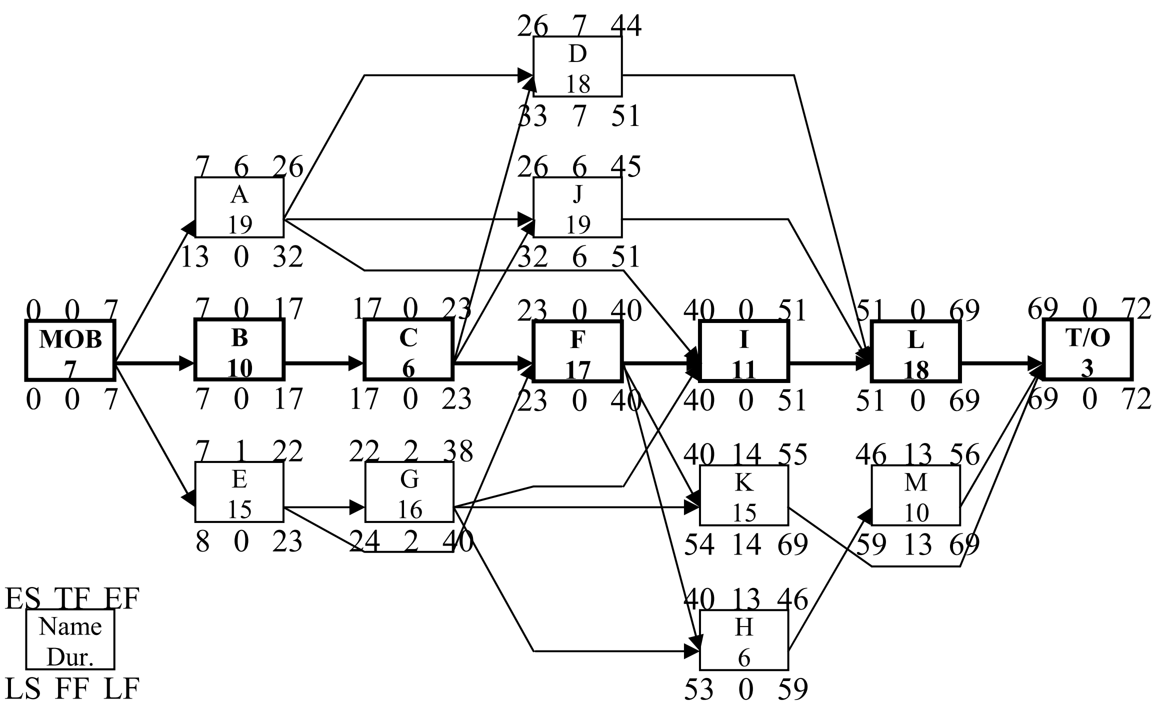
Table 3.4: Exemplar Inputs – Cost and Schedule Activity List with CPM Calculation Results

Activity	Cost	Duration (days)	Successor	Early Start (ES)	Late Start (LS)	Early Finish (EF)	Late Finish (LF)	Total Float (TF)
Mobilization	**$50,000**	**7**	**A, B, E**	**0**	**0**	**7**	**7**	**0**
A	$285,000	19	D, I, J	7	13	26	32	6
B	**$145,000**	**10**	**C**	**7**	**7**	**17**	**17**	**0**
C	**$25,000**	**6**	**D, F, J**	**17**	**17**	**23**	**23**	**0**
D	$210,000	18	L	26	33	44	51	7
E	$150,000	15	F, G	7	8	22	23	1
F	**$195,000**	**17**	**H, I, K**	**23**	**23**	**40**	**40**	**0**
G	$200,000	16	H, I, K	22	24	38	40	2
H	$100,000	6	M	40	53	46	59	13
I	**$110,000**	**11**	**L**	**40**	**40**	**51**	**51**	**0**
J	$250,000	19	L	26	32	45	51	6
K	$255,000	15	T/O	40	54	55	69	14
L	**$310,000**	**18**	**T/O**	**51**	**51**	**69**	**69**	**0**
M	$190,000	10	T/O	46	59	56	69	13
Turn Over	**$25,000**	**3**	**N/A**	**69**	**69**	**72**	**72**	**0**
Total	$2,500,000	72 days						

Boldface activities are on the critical path.
Contractual duration: 90 days
Adopted from Lucko (2005) [costs added]

Summarized in Table 3.4 are the critical path elements found in the literature, and within Figure 3.3 is the graphical portrayal of the network logic, to which activity costs have been added representing those that could be expected of a small to mid-sized project of any type (construction-based projects not limiting). Beyond the cost data, further definition of project constraints is necessary for the exemplar to be sufficient for use herein and overcome an intrinsic shortcoming of critical path network schedules: the CPM duration is limited to the activities contained therein, not to other extraneous conditions placed upon the network. The contractual and other obligations or complexities defining the totality of the project work and duration are not seen beyond the 72-day CPM duration. Therefore, beyond addition of cost constraints to the exemplar, the overall contractual completion of the project is set at 90 days. This yields 18 days of float available for allocation.

Figure 3.3: Exemplar CPM Network Schedule Diagram (Lucko 2005)



3.4.3 Localized vs. General Total Float

Total float (TF) (the time by which an activity may be delayed or extended without affecting the total project duration, or violating a target finish date), requires further definition. Given the addition of 18 days to the 72-day CPM duration, the resulting total float occurs in two distinct types: *general* and *local* 'post-CPM duration float.' 'General post-CPM duration float' relates to the overall project completion (hereinafter identified as 'contract float'), and 'local post-CPM duration float' relates to that within the CPM network and to specific adjacent activities respectively. These need to be identified and addressed separately as one has value to the critical network members (those on the critical path) and the other does not.

3.4.4 Float Allocation Metrics

3.4.4.1 Contract Float Allocation

The addition of activity costs and contract float (CF) to the exemplar results in parallel paths for float allocation. The first is based on activity cost and its square root, and the second is based on duration and duration square root. However, initial analysis of a preliminary contract float model concluded that its allocation is only applicable to critical participants (those activities residing and/or forming the critical path). In this analysis this limits allocation of contract float to activities *Mob*, *B*, *C*, *F*, *I*, *L*, and *T* (shown in bold in the tables and figures). This limited application is due to total float being present to varying extents in non-critical activities that can be expended without consequence to the overall schedule or to the critical activities on parallel or adjacent CPM paths. Calculations herein focus solely on the allocation of contract float to critical activities.

3.4.4.2 Calculation Normalization

The use of disparate elements in calculations, i.e. cost divided by duration and that of partial and incomplete arrays, results in percentages that do not add to 100% of their respective category. They require normalization such that each sum equals 100% and can be accurately compared to each another. To accomplish this, each element of the calculation is multiplied by the reciprocal of the sum of all elements in the category.

Table 3.5: Exemplar Calculation – Duration-Based Normalization

Input Data			Duration Square Root-Based Allocation	
Activity	**Duration**	**Duration Square Root**	**Percent of Contract Float (CF)**	**Normalized Percent of Contract Float (CF)**
Mob	**7**	**2.65**	**14.70%**	**12.21%**
A	19	4.36		
B	**10**	**3.16**	**17.57%**	**14.59%**
C	**6**	**2.45**	**13.61%**	**11.30%**
D	18	4.24		
E	15	3.87		
F	**17**	**4.12**	**22.91%**	**19.03%**
G	16	4.00		
H	6	2.45		
I	**11**	**3.32**	**18.43%**	**15.30%**
J	19	4.36		
K	15	3.87		
L	**18**	**4.24**	**23.57%**	**19.58%**
M	10	3.16		
T / O	**3**	**1.73**	**9.62%**	**7.99%**
Total		51.99	120%	100%
		Standard Deviation		4.17%

Boldface activities are on the critical path.

For example, when calculating the square root-based contract float allocation to critical activities, the Mobilization (*Mob*) activity duration of 7 days yields a square root of 2.65 days that translates to 14.70% of the 18-day contract float duration. Likewise, activity *B* yields 17.57% of the 18 days. The total of all critical activities reaches 120%. Multiplying each activities resulting percentage by 1/120% (or dividing the resulting percentage by the sum of all elements) normalizes the resulting percentages. Presented in Table 3.5 is the normalization process for the duration square root-based array.

3.4.4.3 Measure of Dispersion

The common measures of central tendency used in statistical analysis are for the purpose of estimating the normal values of a data set or population. They are of particular importance in describing the spread of the data around its expected value or around a mean value (the numeric average). Of importance to this research is the variability of the individual calculation arrays, as two or more arrays may have the same mean value but are differently dispersed about that value.

The measure of dispersion most germane to the posited allocation methodology is the standard deviation, as it quantifies the variation of data elements or their scattering from the mean value. In similar fashion to the Penrose Law, it is a square root based derivative of the statistical variance: the sum of the distance of a set of data points from their mean. In this analysis, a lower standard deviation indicates that the data points, the individually allocated distributed float (DF) (the allocated percent of contract float assigned to each critical activity) tend to be closer to the mean and closer to each other. Thus, they represent a *fair* allocation across the critical activities.

3.4.5 Float Allocation Model via the Penrose Square Root Law

3.4.5.1 Distributed Float Calculations

Tables 3.6a and 3.6b depict the exemplar outputs for the cost-based and duration-based contract float allocations respectively (with the best performing array in bold font). The tables present the relevant float metrics developed from the normalization process. To arrive at the float assigned to each critical activity (hereinafter identified as distributed float (DF)), four calculation arrays were developed: cost-based and duration-based arrays together with their respective square roots.

The normalized duration square root-based array begins with the normalized percent of contract float (CF), in this case, 12.21% for the *Mob* activity. This percent is the basis for assignment of the respective distributed float (DF), the 12.21% of the 18-day contract float. For the *Mob* critical activity this equates to a distributed float of 2.20 days.

Returning to the integer rounding precepts of Sainte Laguë and d'Hondt, the 2.20 days are rounded based on *zero digit* rounding to the right of the decimal point. This results in whole day allocations, as partial day activities are anathema to critical path calculations and network schedule systems.

Equation 3.4 represent the translation of the distributed float (DF) allocation process derived from the Penrose Square Root Law into a formula.

$$DF_i = \left\| \frac{\sqrt{AD_i}}{CF} \cdot \frac{1}{\sum_{x=i}^{j} \frac{\sqrt{AD_i}}{CF}} \cdot CF \right\| \qquad \text{[Eq. 3.4]}$$

which simplifies to Equation 3.5:

$$DF_i = \left\| \frac{\sqrt{AD_i}}{\sum_{x=i}^{j} \frac{\sqrt{AD_i}}{CF}} \right\| \quad \text{[Eq. 3.5]}$$

where

AD_i = duration for critical activity i

CF = project / schedule system contract float

DF_i = distributed float for critical activity i

j = number of critical activities (population)

$\| \quad \|$ = denotes rounding function to nearest integer

Table 3.6a: Exemplar Outputs – Cost-Based Calculations and Analysis

Input Data			Cost-Based Analysis			Cost Square Root-Based Analysis		
Activity	**Activity Cost**	**Cost Square Root**	**Normalized Percent of Contract Float (CF)**	**Allocated Contract Float (CF)**	**Integer Rounded Allocation: Distributed Float (DF)**	**Normalized Percent of Contract Float (CF)**	**Allocated Contract Float(CF)**	**Integer Rounded Allocation: Distributed Float (DF)**
Mob	$50,000	$223.61	5.81%	1.05	1	9.94%	1.79	2
A	$285,000	$533.85						
B	$145,000	$380.79	16.86%	3.03	3	16.92%	3.05	3
C	$25,000	$158.11	2.91%	0.52	1	7.03%	1.26	1
D	$210,000	$458.26						
E	$150,000	$387.30						
F	$195,000	$441.59	22.67%	4.08	4	19.62%	3.53	4
G	$200,000	$447.21						
H	$100,000	$316.23						
I	$110,000	$331.66	12.79%	2.30	2	14.74%	2.65	3
J	$250,000	$500.00						
K	$255,000	$504.98						
L	$310,000	$556.78	36.05%	6.49	6	24.74%	4.45	4
M	$190,000	$435.89						
T / O	$25,000	$158.11	2.91%	0.52	1	7.03%	1.26	1
Total	$2,500,000	$5,834	100%	18	18	100%	18	18
	Standard Deviation		12.13%			6.70%		

Boldface activities are on the critical path.

90-day overall contractual completion assumed, leaving an additional 18 days of CF.

Table 3.6b: Exemplar Outputs – Duration-Based Calculations and Analysis

Input Data			Duration-Based Analysis			**Duration Square Root-Based Analysis**		
Activity	Activity Duration	Duration Square Root	Normalized Percent of Contract Float (CF)	Allocated Contract Float (CF)	Integer Rounded Allocation: Distributed Float (DF)	**Normalized Percent of Contract Float (CF)**	**Allocated Contract Float (CF)**	**Integer Rounded Allocation: Distributed Float (DF)**
Mob	7	2.65	9.72%	1.75	2	**12.21%**	**2.20**	**2**
A	19	4.36						
B	10	3.16	13.89%	2.50	3	**14.59%**	**2.63**	**3**
C	6	2.45	8.33%	1.50	2	**11.30%**	**2.03**	**2**
D	18	4.24						
E	15	3.87						
F	17	4.12	23.61%	4.25	4	**19.03%**	**3.42**	**3**
G	16	4.00						
H	6	2.45						
I	11	3.32	15.28%	2.75	3	**15.30%**	**2.75**	**3**
J	19	4.36						
K	15	3.87						
L	18	4.24	25.00%	4.50	5	**19.58%**	**3.52**	**4**
M	10	3.16						
T / O	3	1.73	4.17%	0.75	1	**7.99%**	**1.44**	**1**
Total	72 days	52 days	100%	18	20	**100%**	**18**	**18**
Standard Deviation			7.76%			**4.17%**		

Boldface activities are on the critical path.

90-day overall contractual completion assumed, leaving an additional 18 days of CF.

Tracing a single calculation from beginning to end, the duration square root-based *Mob* activity calculation follows the following sequence: the 7-day activity duration has a square root of 2.65 days, which represents a 'hard' 14.70% of the 18 days of contract float, normalized to 12.21% of contract float (CF), translated to a percentage allocated 2.20 days of float and 2 days of integer rounded distributed float (DF). Cost based arrays follow a similar pattern with the cost and cost square root divided by the 18 days contract float (CF) total yielding the 'hard' or non-normalized allocated percent of contract float. The balance of the calculation remains the same as that previously described and is not cost or duration specific.

3.4.5.2 Calculation Analysis

Viewing the dispersion of the distributed float (DF) as the measure for determining the applicability of cost or duration-based modeling and whether the application of the Penrose Square Root Law makes a material difference in the allocation of contract float (CF) to critical activities, it is apparent that the use of a duration square root based calculation presents the smallest dispersion (the lowest standard deviation) at 4.17%. Likewise, its array of allocation percentages maintains the tightest range: 7.99% to 19.58% (an 11.59% spread) vs. 2.91% to 36.05% (a 33.14% spread) for nominal cost-based calculations that represent the greatest dispersion.

Table 3.7: Exemplar Analysis – Measure of Dispersion Summary (Standard Deviations)

	Nominal Value		Square Root	
Cost-Based (σ)	12.13%		6.70%	
Range	2.91%	36.05%	7.03%	24.74%
Total Distributed Float (DF)	18 days		18 days	
Duration-Based (σ)	7.76%		**4.17%**	
Range	4.17%	25.00%	**7.99%**	**19.58%**
Total Distributed Float (DF)	20 days		**18 days**	

Presented within Table 3.7 are the results with the addition of the integer rounded distributed float (DF) resulting from each array. Despite having a tighter range and standard deviation than that of the nominal cost-based calculation, the nominal duration-based array produces an integer rounded distributed float (DF) total of 20 days, two days in excess of the 18 days of available contract float (CF). This exemplifies the conundrum faced by Sainte Laguë and d'Hondt, and further strengthens the Penrose precepts for the appropriateness of square root-based allocation model. In further support of the square root calculation consideration, the resulting distributed float (DF) quantities are presented side by side in Table 3.8, showing that the Penrose approach yields tighter allocations, which may represent fairer or more even allocations.

Table 3.8: Exemplar Summary – Distributed Float

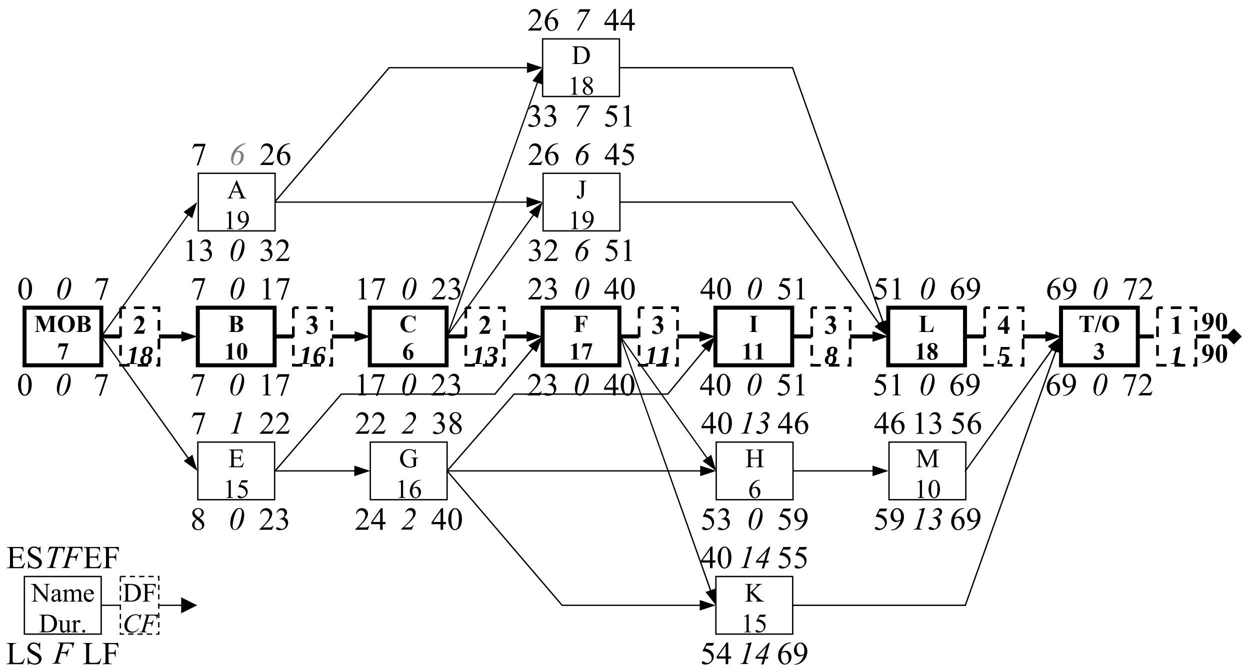
Activity	Distributed Float Quantities			
	Nominal (Full) Value		Square Root (Penrose) Value	
	Cost-Based	Duration-Based	Cost-Based	Duration-Based
Mob	1	2	2	**2**
B	3	3	3	**3**
C	1	2	1	**2**
F	4	4	4	**3**
I	2	3	3	**3**
L	6	5	4	**4**
T/O	1	1	1	**1**
Total	18	20	18	**18**

Boldfaced quantities are the recommended/ 'best fit' allocations

3.4.5.3 Application to Schedule Network Diagrams

Building upon this analysis, this research proposes the inclusion of the newly characterized distributed float (DF) and contract float (CF) within critical path network diagrams. Depicted within Figure 3.4 is the inclusion of such float metrics, and more specifically the values for the recommended allocation based on duration square root. When including such allocations, the top number in the additional box (formed with dashed lines) associated with the critical activities represents the distributed float (DF) for the activity, while the bottom number shown the remaining contract float (CF). In keeping with the legal precept that float is a fleeting or diminishing quantity overcome by time, contract float (CF) decreases by the distributed float (DF) quantity of the predecessor activity.

**Figure 3.4: Exemplar CPM Network Schedule Diagram
Including Distributed and Contract Floats**



3.4.6 Banzhaf Considerations

Returning to the second analytical voting model, the Banzhaf Power Index, its applicability to the float allocation model under consideration is not as direct as that of Penrose. Whereas a Penrose analogy facilitates an allocation method, a corresponding Banzhaf analogy would predicate an index focusing on the expenditure of distributed float (DF), not its equitable allocation. Accordingly, development of such an index though of merit, is beyond the capabilities of this exemplar and not further developed in this research.

3.5 Application

3.5.1 Construction Duration Variability

Construction projects, and in particular critical infrastructure projects, are notorious for time and cost overruns (Creedy et al. 2010, Shane et al. 2009, Kim 2007, Georgy et al. 2000). The overruns are in part due to a "deficiency in managing the scope, time, quality, cost, [and] productivity" (Jergeas and Ruwanpura 2010, p.40). Depending on project type, schedule variation ranges from early finishes to far exceeding planned durations. An international study of over 200 building projects portrayed that approximately one-third finished on or ahead of planned duration (Acharya et al. 2006), with 20% exceeding their planned duration by more than 50% (Table 3.9 depicts the full schedule variability findings of Acharya et al.). Bhargava et al. (2010) found that for about 90% of projects in a 1,800-plus highway construction project study, the actual construction durations exceeded the planned construction duration. In some instances, actual construction duration exceeded that expected by a factor of five. Table 3.10 identifies the full array of schedule durations.

Table 3.9: Building Project Duration Delay (Acharya et al. 2006, Table 4)

Schedule Delay	Percent of Projects
Early Finish	5.5%
No Delay	28.9%
Under 10%	7.0%
11 - 25%	14.8%
25-50%	23.4%
Over 50%	20.3%

Table 3.10: Highway Construction Project Durations (Bhargava et al. 2010)

Percent of As-Planned Schedule	Percent of Projects
Early Finish	11.12%
100 - 200%	43.29%
201 - 300%	16.70%
301 - 500%	19.28%
501 - 1000%	8.27%
Over 1000%	1.34%

Project management is doubly challenged, as "poor scheduling and control" are contributing causes for these overruns (Akpan and Igwe 2001, p.367, Jergeas and Ruwanpura 2010), and then managers remain challenged to mitigate their combined impact. The application of a float allocation model represents a vehicle to overcome these challenges and maximize the flexibility available to critical activities presented by distributed float. To demonstrate this, duration variation is applied to the exemplar via Monte Carlo simulation, which will form the data for statistical analysis to support the hypothesis of the exemplar.

3.5.2 Probability Distribution Function Selection

Within construction industry simulation modeling, the appropriate probability distribution formula (PDF) to employ is the seminal decision. A multiplicity of PDFs are suitable for use in construction industry modeling (Arízaga 2007), including the normal and lognormal distributions (Touran 1997), the beta distribution (Touran 1997, Fente et al. 2000, Maio et al. 2000, Schexnayder et al. 2005) and the triangular distribution (Back et al. 2000, Arízaga 2007). There were not significant differences in the simulation outputs experienced with the use of beta versus triangular distributions on ground operations (Wilson et al. 1982).

Herein, considering the findings of Raymond (1999), the triangular PDF will be employed to vary schedule constraints. He concluded that (1) the triangular PDF is reasonable for describing the risk and/or uncertainty for a task duration, (2) the structure of the PDF fits with construction project management (minimum expectation, most likely occurrence (the plan best case), and the maximum expectation, (3) it is simple, intuitive, easy to understand, and easily employed in mathematical models within Monte Carlo analysis, and (4) the use of other more complex distributions provides little gain in modeling results.

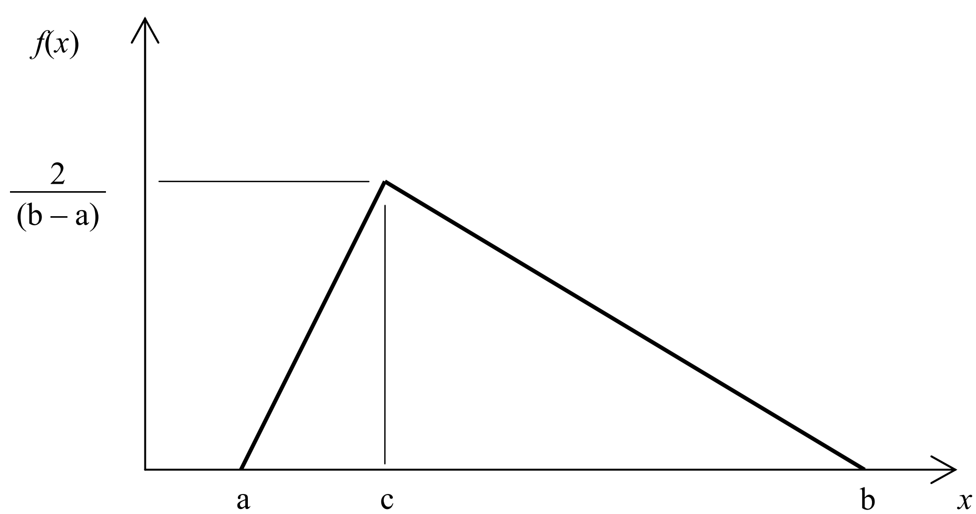
The sole detraction or disadvantage to the use of the triangular PDF becomes the primary reason / justification for its use in construction industry simulation: there is an over-emphasis of the outcomes in the direction of the triangular skew (where the area of the triangle is predominantly on one side of the mode value). This mimics the one-third – two-thirds distribution results from the Acharya et al. (2006) study of project durations summarized in Table 3.9.

The triangular PDF is defined by three equations representing two sloped lines and one point, Equation 3.6 as depicted in Figure 3.5:

$$f(x|a,b,c)=\begin{cases} \dfrac{2(x-a)}{(b-a)(c-a)} & a \le x \le c \\ \dfrac{2}{(b-a)} & x = c \\ \dfrac{2(b-x)}{(b-a)(b-c)} & c < x \le b \end{cases}$$

where $x < a = 0$ and $x > b = 0$ [Eq. 3.6]

Figure 3.5: Graphical Representation of Triangular Distribution



3.5.3 Exemplar Distribution Calculations

In addition to selection of the specific PDF, additional elements are required to implement a Monte Carlo simulation of the exemplar schedule network. The primary concern are the *a* and *b* components required for the distribution, the lower and upper bounds of the distribution respectively, and *c*, the mode value, herein defined as the as-planned or expected duration. The relationship between these components is demonstrated by the probability expectation for the occurrence of *c*, the mode value, defined by Equation 3.7.

$$P(x|c): \frac{2}{(b-a)} \qquad \text{[Eq. 3.7]}$$

Calculation of the individual PDF bounds (the range of the function, *a* and *b*), requires rounding, because partial days are anathema to network schedule systems. The results for employing the triangular PDF will be rounded to integers to represent whole days (by necessity from accepted scheduling convention and software limitations) by the using numeric (decimal) rounding rules to zero decimal places, due to the triangular function being a continuous distribution where any value between *a* and *b* is possible.

3.5.4 Monte Carlo Simulation and Statistical Bootstrapping

To provide further insight into the interactions of the exemplar schedule participants under the aforementioned conditions of uncertainty and variability, a Monte Carlo simulation (or simulation) will be performed. Absent a vast array of network schedules with common participants (activity ownership) against which to compare as-planned versus actual durations and determine the need for float, a method to approximate a cohort of projects (to simulate the variety of results likely for the exemplar) is to apply PDF values to the exemplar schedule for which multiple iterations are then produced and analyzed as a cohort of projects.

To generate the requisite data population, statistical bootstrapping will be used, as the sample size is insufficient for straightforward statistical inference. Bootstrapping is generally useful for estimating the distribution of a statistic (e.g. the mean, variance, etc.) when normal theory is unavailable to help estimate the statistical distribution. The statistical distribution herein is the resulting activity durations of a single network schedule system. To bootstrap data, the "data-based simulation method for statistical inference" (Efron and Tibshirani 1993, p.5), is to create an initial population of data (herein the network schedule activity durations) and then through drawing and replacement (herein individual iterations of the simulation model), a bootstrap population of data is derived.

The purpose of using Monte Carlo simulation is to bring variability and uncertainty to the activity durations within the exemplar network schedule system and gauge the performance of the activities forming the critical path with respect to the need and quantity of distributed float. The activity durations generated by the simulation model will serve as the bootstrapped data for the calculation of the delta to the as-planned durations in the benchmark schedule (the original exemplar schedule) and whether that exceeds the distributed float presented herein for the critical activities: *Mob*, *B*, *C*, *F*, *I*, *L*, and *T/O*. It is expected that this will provide insight into the sufficiency of square root-based distributed float allocation in the exemplar.

3.5.5 Monte Carlo Simulation Model

The Monte Carlo simulation model used to bootstrap a statistical population was developed using the @Risk™ (At-Risk) Risk Analysis and Simulation Add-In Program, Version 5.7 (September, 2010) for Microsoft® Excel (2007), from the Palisade Corporation, Ithaca, NY.

To create the working model representing the exemplar network schedule system, an activity data and calculation box representing the schedule components for each activity and the allocated distributed float was developed and positioned within the Excel worksheet in relative position to that depicted in the exemplar CPM network diagram (Figure 3.4). Excel "MAX" statements set the predecessor-to-successor logic for the early start (ES) of each activity (with the maximum being the maximum schedule duration taken by the predecessor activities requiring completion before the successor may begin). Early finish (EF) duration is calculated by adding the duration of the subject activity to the early start. This is completed for all activities, such that the schedule forward pass is complete. A backward pass is then completed using only "EQUAL" statements from activity to activity upon subtraction of the activity duration from the late finish (LF) to create the early finish (EF). A portion of the model is depicted in Figure 3.6. The activity and data and calculation box for activity *L* is deconstructed in Figure 3.7, including the @Risk PDF parameters and output expectations.

Figure 3.6: Excel-Based @Risk Simulation Model – Partial Segment

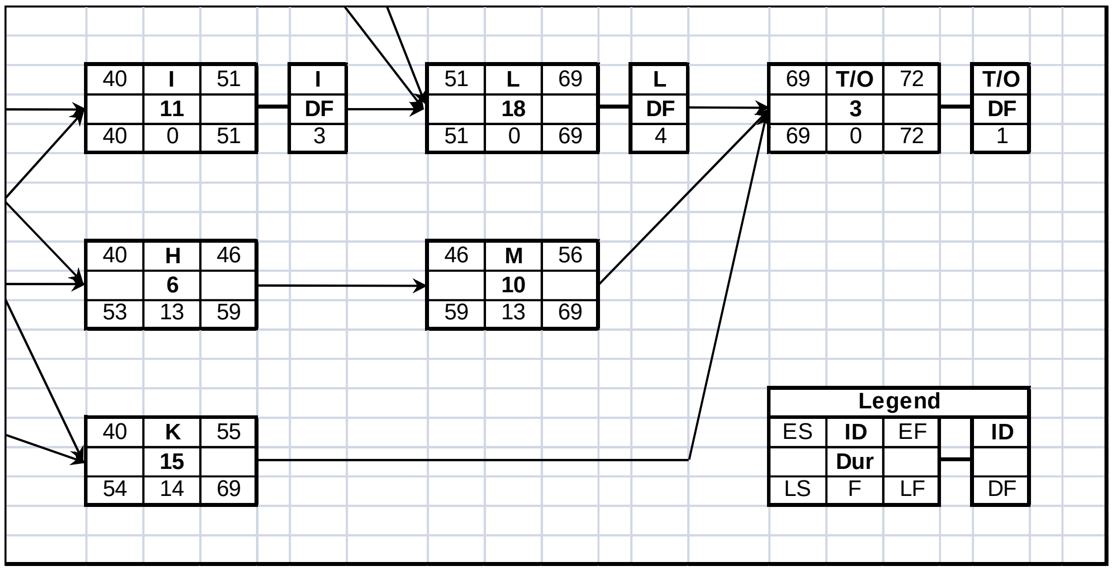
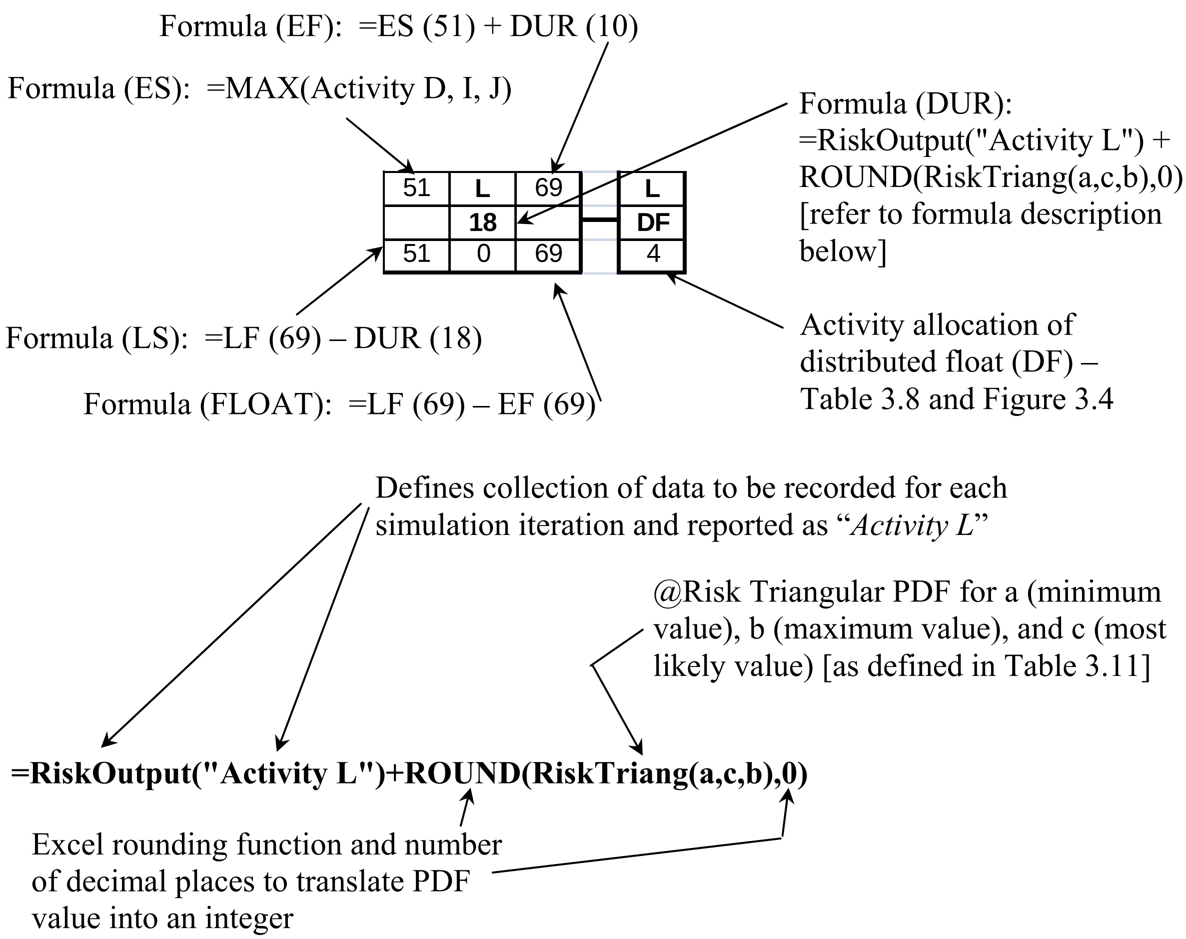


Figure 3.7: Data and Calculation Box Deconstruction – Activity *L*



3.5.6 Probability Distribution Function and Model Calibration

To arrive at the baseline PDF to be used in the Monte Carlo simulation, activity durations were varied as a percent function of the as-planned duration and analyzed against the idealized one-third – two-thirds Acharya et al. (2006) study duration distribution versus the 72-day expected duration. The Excel table used to generate the 'best fit' PDF values and populate the duration values for each activity data and calculation box is depicted in Tables 3.11a and 3.11b, where *a* = minimum value, *b* = maximum value, and *c* = most likely value.

Table 3.11: Monte Carlo Simulation Activity Duration PDF Generator:

Array A – Best Fit Values

Triangular PDF			
Percent < APD			**25%**
Percent > APD			**28.5%**
ID	**a**	**c**	**b**
Mob	5	*7*	9
A	14	*19*	24
B	8	*10*	13
C	5	*6*	8
D	14	*18*	23
E	11	*15*	19
F	13	*17*	22
G	12	*16*	21
H	5	*6*	8
I	8	*11*	14
J	14	*19*	24
K	11	*15*	19
L	14	*18*	23
M	8	*10*	13
T/O	2	*3*	4

APD = As-Planned Duration

Array B – Intuitive Values

Triangular PDF			
Intuitive Values			
ID	**a**	**c**	**b**
Mob	3	*7*	7
A	14	*19*	21
B	6	*10*	18
C	3	*6*	9
D	13	*18*	29
E	12	*15*	24
F	10	*17*	26
G	11	*16*	26
H	3	*6*	11
I	5	*11*	17
J	13	*19*	30
K	11	*15*	24
L	15	*18*	18
M	8	*10*	14
T/O	2	*3*	5

A second intuitive approach to schedule durations was developed to better represent the way in which a construction project represented by the exemplar network system would perform. In this version, the activity durations are based upon intuition for activity level duration variation from that planned and consider elements such as sequence in the schedule, the level of parallel activity, comparative duration, and the potential for acceleration. The characterizations for the critical activities of the exemplar are depicted in Table 3.12. The skew and potential for acceleration are dependent upon the activity characteristics in the exemplar CPM Network diagram (Figure 3.4).

Table 3.12: Intuition-Based Simulation Critical Activity PDF Values and Characterizations

Probability Distribution Function Values					Skew	Position	Parallel Activity	Duration	Probability for Success	Acceleration Potential
ID	**a**	**c**	**b**	**P(x|c)**						
Mob	3	7	7	0.500	Left	Early	Low	Short	High	Yes
B	6	10	18	0.167	Right	Early	High	Medium	High	No
C	3	6	9	0.333	None	Early	High	Short	High	No
F	10	17	26	0.125	Right	Middle	High	Long	Medium	No
I	5	11	17	0.167	None	Middle	High	Short	Low	Yes
L	15	18	18	0.667	Left	Later	Low	Short	High	Yes
T/O	2	3	5	0.667	Right	Later	Low	Short	High	No

Calibration and validation results for both modeling approaches are shown in Table 3.13.

Table 3.13: Simulation Model Combined Calibration and Validation Results

Model Calibration			**Model Validation**			
Activity Durations			**Performance (72-day Duration)**			
Less than APD*	**Greater than APD**	**Model Iterations**	**Less than APD**	**Greater than APD**	**Max (days)**	**Min (days)**
25%	**28.5%**	**100,000**	**33.7%**	**66.3%**	**87**	**62**
Intuitive		**100,000**	**30.6%**	**69.4%**	**92**	**59**

* APD = As-Planned Duration

3.5.7 Analysis of Simulation Distributed Float Requirements

The results for the simulations depict the same critical activities within the exemplar, their need for distributed float (DF), and are presented in Table 3.13. Multiple 100,000 iteration simulations were run (to provide a sufficiently large statistical population) beginning with the best fit duration PDF values of 25% below the most likely activity duration (PDF component *c*) and 28.5% above the most likely value (the combination meeting the Acharya et al. (2006) idealized duration distribution), and expanding the distribution range to reach 90% below / 200% above the most likely activity duration (90% selected as the lowest duration as all activities require time; the 90% reduction defaults to minimum 1-day duration). The need for distributed float (DF) beyond that allocated under the exemplar model is identified in Table 3.15.

These values resulted from a simple counting mechanism for each simulation iteration whereby a value of "one" was recorded for each activity when the difference between the PDF duration and the as-planned duration exceeded the distributed float (e.g. activity *L* PDF duration = 21, minus the as-planned duration = 18, yields 3 days float requirement, which is greater than the 2 days of DF, and results in a 1 being recorded for the iteration), and a value of "zero" was recorded when the activity PDF duration less the as-planned duration is less than the allocated distributed float. This was accomplished by way of Excel "IF" statements as depicted within Figure 3.8.

Figure 3.8: Distributed Float Need Counting Mechanism

Critical Activities	
ID	**DUR<=APD**
Mob	0
B	1
C	1
F	0
I	0
L	1
T/O	0
1 = Needs Additional DF	

Formula (Additional DF Count):
=RiskOutput("Activity L Greater Than APD + DF")+ IF(PDF Duration – APD > DF,1,0) [refer to formula description below]

Defines collection of data to be recorded for each simulation iteration and reported as "*Activity L Greater Than APD + DF*"

Excel 'IF' function condition: Value of activity-specific distributed float

=RiskOutput("Activity L Greater Than APD + DF")+IF(PDF Duration – APD > DF,1,0)

Excel 'IF' function condition: difference between PDF duration value and as-planned duration

Excel 'IF' function: 1 = additional float beyond allocated DF required, 0 = no additional float required

Table 3.14: Simulation Model Results – Distributed Float Sufficiency

Activity Durations		Completed within Duration + Distributed Float (DF)						
Less than APD*	**Greater than APD**	**Mob**	**B**	**C**	**F**	**I**	**L**	**T/O**
25%	**28.5%**	**100%**	**100%**	**100%**	**95.00%**	**100%**	**99.44%**	**100%**
30%	30%	100%	100%	100%	99.50%	100%	99.5%	100%
50%	50%	91.96%	95.50%	98.61%	80.23%	90.53%	87.50%	95.83%
90%**	100%	77.75%	77.76%	81.44%	66.50%	75.65%	70.22%	85.00%
90%	125%	68.70%	68.44%	70.91%	59.49%	67.19%	61.85%	73.96%
90%	150%	61.36%	63.26%	66.47%	52.51%	60.29%	56.40%	65.00%
90%	200%	52.77%	53.06%	55.76%	44.16%	51.39%	47.00%	57.81%
Intuitive		**100%**	**78.91%**	**98.61%**	**78.99%**	**91.32%**	**100%**	**95.83%**

* APD = As-Planned Duration

** 'Less than APD' percentages become problematic above 66%, as activity T/O rounds to zero. This requires an override via the establishment of a 1-day minimum duration.

The bootstrapped data for activity and project durations presented within Tables 3.14 and 3.15 was derived by translating the individual as-planned durations were into a triangular PDF by multiplying the as-planned duration by one-plus or one-minus a percentage for the *b* and *a* PDF elements respectively (e.g. the 30% / 30% activity duration simulation results per Table 3.14) were created by multiplying each as-planned duration by 1 – 30% for the lower duration and 1 + 30% for the upper duration). The resulting durations were rounded to become integers (zero decimal places) to represent whole days. This process was repeated at varying percentages to represent major changes in the duration range (typically in 50% increments, with the exception being the lower PDF value being limited to 90% and a default value of 1-day). The 90% / 125% model variation was specifically crafted to yield a total duration as close as possible to the 90-day contractual limitation.

The data lends credibility to the supposition of this research. Distributed Float allocated based on the square root of critical activity duration provides flexibility to critical activities and as demonstrated by the 25% / 28.5% model variation results in nearly all activities being completed within the as-planned duration plus the allocated distributed float. Less than one percent of the activity iterations (approximately 5,500 instances out of 700,000 possible critical activity instances) required float beyond that allocated, and in all cases the project was completed within the hypothetical 90-day contractual requirement. As for the intuitive model version, additional distributed float was required by only eight percent of the activity iterations, and with the exception of 0.012% of the iterations (1,200 out of 100,000) did the schedule duration exceed the 90-day requirement.

**Table 3.15: Simulation Model Results –
Need to Acquire Additional Distributed Float**

Activity Durations					Need for Additional Distributed Float (DF)
Less than APD*	Greater than APD	Range		Average Completion	All Activities Combined
		Min	Max		
25%	**28.5%**	**62**	**87**	**74**	**0.79%**
30%	30%	60	86	73	0.71%
50%	50%	54	101	76	8.55%
90%	100%	45	124	82	23.67%
90%	125%	45	140	90	32.78%
90%	150%	52	154	98	39.24%
90%	200%	49	178	112	48.29%
Intuitive		**59**	**92**	**75**	**8.05%**

* APD = As-Planned Duration

It is not until the more extreme variations of the PDF that the 90-day schedule requirement is jeopardized and the requirement for distributed float (DF) beyond that allocated becomes substantial. The 90% / 125% model variation reaches as an average the 90-day contractual duration requirement while requiring additional distributed float by nearly one-third of the activity iterations.

It can be concluded that the allocated distributed float (DF) as determined by activity duration square root facilitates completion within the contractual expectations of the exemplar up to the point of a 100% schedule delay (actual completion being twice the as-planned duration) and well within the preponderance of expected durations depicted by Acharya et al. (2006) and Bhargava et al. (2010); while requiring float in excess of that allocated in less than one-third of the time.

By extension it can also be deduced that the inclusion of distributed float is an efficient approach to network schedule system operations and management. It affords critical activities flexibility, creates the potential for a float exchange market, while negligibly impacting overall project duration (as exemplified by the two and three days of additional average time to complete the exemplar network for the 15% / 28.5% and Intuitive model variation per Table 3.15).

3.6 Conclusions

This research began with decision-making models and voting practices as measures of risk within network schedule systems and the expectation of an improved methodology for allocating it among a network schedule system's individual activities. Through the literature, float was confirmed as the measure of risk in network schedules and that float, and in particular total float, is a diminishing commodity, has no direct ownership or control, and is expended by the entity/activity of first want. Through the exemplar method, it has been demonstrated that the duration square root allocation model is superior to that of the nominal duration and superior to that of both cost-based models, given the tighter dispersion of its allocation percentages.

Table 3.16: Ubiquitous Float Characteristics within a Decision-Making Allocation Model

Float Characteristic	Decision-Making Allocation Model Application
Diminishing Commodity	Distributed float (DF) diminishes as critical activities conclude (Figure 4). Future research will consider/posits that DF may be traded as a commodity
No Direct Ownership or Control	The allocation model distributes contract float (CF) and fosters ownership by those participants in most need – critical activities (those representing the critical path)
First Come, First Served Consumption	By allocating contract float (CF) as distributed float (DF) thereby instituting an ownership atmosphere, FC-FS consumption is negated in favor of consumption directed by time and 'controlling' critical activity

This research concludes that contract float (CF) allocation is only appropriate to critical activities, not to all network schedule system participants. By definition, critical activities are afforded no float within the CPM or network schedule system, thereby relegating total float (when occurring within the as-planned schedule duration, i.e. within the exemplar 72-day duration) to the non-critical activities and rendering the need for allocation of float residing *outside* the network duration to non-critical activities moot. Total float may be present in some or all non-critical activities and can be expended at will without impacting preceding, succeeding, or parallel activities, nor delaying the overall schedule duration. Table 3.16 presents a comparison of the float allocation model to the research expectation.

In addition, this research has contributed two new float characterizations to the body of knowledge:

- *Contract Float:* The difference between the duration of a critical path network schedule duration and the contractual completion time.
- *Distributed Float:* The allocation of contract float (CF) to the critical activities by a mathematical model. Distributed float (DF) cannot exceed contract float (CF).

3.7 Future Research

This new approach to allocating risk (i.e. the expenditure of float) is based upon previously vetted decision-making models and voting practices opens extended avenues for future research. As initially introduced, this research represents the second element in the three-part fulfillment of risk quantification, pricing and mitigation, allocation, and the development of a method for the distribution of total float to where it is most needed (critical activities), is focused on the allocation component. As a valid risk quantification and float allocation model has been crafted via European voting models, future investigation depicting analogous research and extending the aforementioned components of the *Total Float Traded as Commodity* notion of de la Garza et al. (1991), is warranted. In specific, future research resulting from this investigation should focus on the application of the Banzhaf Power Index and its relevance to the consumption of float in network schedule systems.

It is expected that the remaining elements in this three-part research endeavor will complete the components necessary for a float trading means will address risk's predominant location, magnitude and value (price and/or cost) within network schedule systems, and in particular within construction project CPM networks. In similar form to the methods employed herein, subsequent research should engage concepts currently vetted, in existence, or in practice. It is expected that a working predictive modeling mechanism will result from the exploration and analysis of risk's residence in network schedule participants along with a 'market' model for its exchange.

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CHAPTER 4

Real Options as a Model for the Monetization and Consumption of Flexibility within Network Project Schedules

Abstract. Risk exists in all construction projects where it resides among the collection of subcontractors and their array of individual activities. It exhibits itself in different ways in projects of all sizes and complexities. Wherever risk resides, the interaction of participants to one another becomes paramount in the way that risk is viewed and addressed. Within a network project schedule, inherent risk becomes recognizable, quantifiable, and can be mitigated by consuming float – the flexibility of a project to absorb delays. Allocating, owning, valuing, and expending float has been pondered since the inception of the critical path method. This research investigates the final element of a three-part treatise that examines how float can be traded as a commodity, an unrequited concept in construction engineering and management whose promise remains unfulfilled for lack of a holistic approach. Real options theory of finance and capital budgeting (the periodic valuation of the decisions associated with non-derivative based tangible assets) is explored as a vehicle to price allocated float for trading. The binomial decision tree or lattice of real options is analogous to the critical decision points within network schedules. At such points participants may use or trade float to reduce delays along or near a critical path, or reserve it to hedge against future risk. These real option decisions within construction projects function like the put and call options of financial derivative trading. This research represents the seminal theoretical extension required for the valuation of tradable float to develop a trading model across a network schedule system's critical participants.

4.1 Introduction

Where risk and uncertainty reside, flexibility has value (Trigeorgis 1996). Placing a monetary value on flexibility depends on the source of the underlying risk, the point at which risk appears, and the venue to which it belongs. Risk in network schedule systems, and in particular in the construction industry, is most recognizable and quantifiable by its impact on a project's schedule, as measured by consumption of its float (the ability to absorb delays without impacting completion). Valuation is the process of determining by way of an estimate the current worth of an asset (investments in marketable securities such as stocks, options, business enterprises, or intangible assets like patents and trademarks), or of a liability (corporate bonds and other forms of indebtedness) (WebFinance 2011b). But just how valuable is flexibility in scheduling, and in particular, how is a value determined? These decisions are the property of corporate finance and capital budgeting.

This research, the completing element in a three-part approach to modeling how systematic risk can be quantified, priced, diversified and/or mitigated, and the development of a prediction method for where risk is likely to reside, is focused on the pricing flexibility in a construction project's schedule. It is an extension of *Total Float Traded as Commodity* (de la Garza *et al.* 1991) and is rooted in capital budgeting and real options.

4.2 Literature Review

In 1983, the future chairman of the U.S. Federal Reserve posited that the presence of uncertainty can increase the value of delaying decisions (Bernanke 1983). He examined investment decisions subject to two simple assumptions: irreversibility – some investment decisions cannot be undone or substantially changed without incurring great or sometimes prohibitive costs, and new information – not all the information relevant to making a decision may be immediately available, and new or better information may become available in the future.

Bernanke concluded that postponing a decision, while maintaining the ability to commit at a later time, can prove desirable by allowing choice only after important information is revealed. In essence, Bernanke maintained that the right (option) to make a decision in the future has inherent value. This conclusion extends credibility to the primary thesis of options theory and in particular real options in decision-making and valuing flexibility.

This concept is best brought forward by what is considered to be the earliest use of a real option to hedge a future decision. Aristotle (tr. Rackham 1944) tells of the sophist philosopher, Thales of Miletus, and his celestial divination of a bountiful olive harvest. According to Aristotle, Thales observed during the preceding winter that there was going to be a large crop of olives. He raised a small sum of money and paid deposits by which he bought the rights to use later at the usual rate, "the whole of the olive-presses in Miletus and Chios." He was able to secure the exclusive use of all presses "at a low rent as nobody was running him up; and when the season arrived, there was a sudden demand for a number of presses at the same time, and by letting them out on what terms he liked he realized a large sum of money" (Aristotle, Pol. 1.1259a).

While Aristotle was mostly concerned with the philosophical markings surrounding the use of one's intellect to profit and not the attributes of the transaction, it is the transaction that is *apropos* herein. The real option transaction, the Bernanke decision to wait for additional information while avoiding an irreversible decision, can be traced thusly:

- Thales bought the right, but not the obligation to rent the olive presses at a later time – in short, he purchased a call option.
- He contracted for a predetermined price the rental of the presses – the exercise price (of sort).
- The initial transaction – the purchase of the option, was completed in advance of an uncertain future; requiring additional information to determine if uncertain future events would make the underlying asset (the olive presses) more or less valuable.

Had the olive harvest not proved to be bountiful, Thales would not have rented the presses and lost only his initial payment as a sunk cost (the option would have expired out-of-the-money). The early lesson with respect to real options (options surrounding tangible assets), is that:

> The value of the option increases with the level of uncertainty of the underlying variable. The logic is straightforward. If there is no uncertainty over the size of the olive harvest, which is known to be normal, then the market rental value of the presses will also be normal, and Thales' option will be worthless. But if the size of the harvest is uncertain, there is a chance that his option will finish in the money. The greater the uncertainty, the higher the probability that the option will finish in the money, and the more valuable the option (Copeland and Keenan 1998, p.41).

4.2.1 Flexibility, Uncertainty, and Risk

4.2.1.1 Uncertainty

Flexibility is necessitated by uncertainty, and uncertainty generates risk. Uncertainty is defined as the "[l]ack of sureness about someone or something. Uncertainty may range from a falling short of certainty to an almost complete lack of conviction or knowledge especially about an outcome or result" (Oxford University 2010b). Uncertainty is credited with the reason why project planning is difficult and inflexible plans are considered suboptimal (Dowlatadabi and Toman 1990). All too often, uncertainty is regarded as a generic concept for the unknown. However, uncertainty in the context of flexibility relates to the unknown at a given point in time, and is not to be confused with the unknowable (Ku 1995). Schweppe et al. (1989) define uncertainties as quantities or events that are beyond the decision maker's foreknowledge or control.

Uncertainty and risk are often used interchangeably. Knight (1921, p.19) first distinguished between measurable risk and "unmeasurable [sic] uncertainty." Strangert (1977, p.35) interprets Knight as follows: "uncertainty refers to an unstructured perception of uncertainty and risk to the situation in which alternative outcomes have been specified and probabilities been assigned to them." Merrill and Wood (1991) observed the causal relationship between uncertainty and risk: uncertainty refers to factors not under control and not known with certainty, whereas *risk* is a hazard because of *uncertainty*.

Uncertainty can be viewed from numerous perspectives. It can be described positively where the possibility of returns is above average, or negatively when the expected returns are below average (Bernanke 1983). Similarly, uncertainty can be described by the factors governing it as either internal uncertainties or external uncertainties, the former being within organizational control and the latter being beyond organizational control (Ku 1995).

Uncertainty may also be considered short-term or long-term, of which time is an important element. Uncertainty can be resolved with the passage of time as well as by investing in information. It may be quantifiable and normal, or of the non-quantifiable sort, where the distinction between them is often attributed to the amount of control and foreknowledge afforded (Merrill and Wood 1991).

Uncertainty in the pejorative is often equated with risk, defined as someone or something that creates or suggests a hazard, or the exposure to danger or hazard (Oxford 2010a). Merrill and Wood (1991) further that uncertainty is those factors not under control of management and not known with certainty, and risk is the hazard posed because of uncertainty. Risk can also be described actively as the bad consequence of taking action in the presence of uncertainty (Ku 1995), and passively as "the adverse consequence of a firm's exposure to uncertainty" (Amram and Kulatilaka 1999, p.8).

4.2.1.2 Flexibility

Klein (1984) defined the notion of flexibility in two forms, type-I and type-II flexibility, that may be interpreted as the flexibility required to accommodate known uncertainty, i.e. risk, and the unknown uncertainty. Carlsson (1989) furthered Klein's typology by characterizing and assigning type I and type II flexibility as *risk* and *uncertainty* respectively. Type I flexibility surrounds foreseeable events and can be accommodated by internal processes. Type II flexibility is inherent within organizations; it is represented by the risk-taking attitudes of its people, their expectations for change, and their interactions over the long term.

There remains a close relationship between flexibility and uncertainty. They come together and establish that flexibility is valuable when there is uncertainty, and flexibility is a way of coping with uncertainty. However, there is no empirical evidence that flexibility reduces uncertainty (Ku 1995). In fact, overanalyzing and/or overcompensating for flexibility, i.e. continual 'tweaking' of the permutations surrounding the potential decision choices, may create more uncertainty and complexity for the decision maker.

Beyond Klein's taxonomy, flexibility has been defined in multiple ways. Its definitions within the literature remain consistent and reflect that flexibility is the 'potential' or the 'capability to respond to change' (Mandelbaum and Buzacott 1990, Slack 1983). Others (Gupta and Buzacott 1988, Mandelbaum 1978, Eppink 1978, Ansoff 1968) arrive at similar characterizations for the ways in which to respond to change and uncertainty: active (action) flexibility – the ability to respond by changing or reacting, and passive (state) flexibility – the absence of the need to change or react due to some form of immunity, insensitivity or the ability to tolerate the uncertainty.

Passive or state flexibility in practice is also characterized as robustness and does not need new or better information, as passive flexibility already exists when changes have taken place and a new state is entered. Active flexibility is only needed when there is less than sufficient or less than perfect information available to decision-makers and is acquired by taking the appropriate action as uncertainty-driven change takes place and a new state becomes advantageous (Mandelbaum 1978).

The dichotomous characterization of flexibility agrees with the conceptual analysis that flexibility is: the inherent capability to modify, the ability to accommodate, and successfully adapt to changes. The ability to endure such change refers to robustness (Evans 1982). Ku (1995, p.317) posits that flexibility is therefore:

- "The general capacity to deal effectively with the widest range of possibilities.
- The ability to perform well both in the old state before a change and in the new state after the change.
- The ability to switch from the first period position to a second period position at low cost.
- The set of remaining programs after the initial choice has been made.
- The system's ability to perform different jobs that may occur or to perform one job under differing conditions."

Flexibility requires the following elements to be appropriately defined and to address the three conditions under which options (flexibility) are valuable: uncertainty, time dependence, and discretion (the ability to exercise and change) (Kogut and Kulatilaka 1994):

- Flexibility conveys change, usually in the future tense, it is a potential.
- Flexibility signifies more than one way of responding to change; it is a range that includes a number of alternatives.

- Flexibility is different from gradual change; the element of time is very important, which includes responsiveness, lead time, and time to change.
- Flexibility encompasses the conditions and existence of uncertainty, alternatives, and strategies.
- Flexibility inherently connotes favorability; which differentiates between the choices available.

Ku (1995, p.300) adds to the definition of flexibility and differentiates it from robustness by stating:

> [F]lexibility means the ability to change by (quickly) moving to a different state, selecting a new alternative, or switching to a different production level. Robustness, on the other hand, is associated with not needing to change. While flexibility is a state of readiness such as the ability to react to change, robustness is a state of being such as a resistance or an immunity to change. Flexibility and robustness are not opposite or the same, but merely two sides of a coin, corresponding to two ways of responding to uncertainty.

4.2.1.3 Robustness

Flexibility is not to be confused with robustness. Robustness has several definitions, but when linked to flexibility and uncertainty, it represents the ability to satisfactorily endure all of the envisioned risks or contingencies. In financial terms, a robust plan is one whose cost varies little with changes in assumptions (Hobbs et al. 1994). It is a system's ability to cope with change independent from the development of future events. That is, when a system is robust, it becomes insensitive to or tolerant of the expected, but unforeseen states of nature (Ramírez 2002). Flexibility is best defined as the ability of a system to respond to *unforeseen* changes (Evans 1982, emphasis added). However, often, the ability to respond to foreseen and/or expected change is characterized as adaptability.

Of particular consequence is that flexibility and robustness are only valuable in an uncertain environment, and both carry a price. The cost of maintaining a robust approach is largely known and is concentrated in the present. Conversely, the costs associated with flexibility are uncertain and are for the most part deferred to the future. Not only are these costs differentiated by their timing, their origin and natures differ accordingly. Costs associated with robustness can be found in the opportunity cost of excess capacity (over building / over designing), while the costs associated with flexibility are of the insurance type, e.g. redundancies, spare parts, etc. "In general, the costs of robustness are not offset by any gains in project value, since robustness does not take advantage of the value generated by uncertainty, as does flexibility" (Ramírez 2002, p.30).

4.2.1.4 Valuing Flexibility and Uncertainty

"The value of flexibility is a function of variation in price and how well that variation can be predicted before the decision is made" (Marschak and Nelson 1962, p.52). It is a function of the specific uncertainty with which flexibility deals and the quality of information regarding the uncertainty. Based on this relationship, the greater the uncertainty, the greater the value of flexibility (Ku 1995).

These flexibility types correspond to two manners of responding to uncertainty. Passive flexibility (robustness) allows uncertainty to be ignored. Nothing needs to change; it facilitates the state of being insensitive to external stimuli. Conversely, active flexibility connotes the ability to change when faced with external stimuli. Finance addresses the ability to accommodate uncertainty and flexibility, and the risk attributed thereto, as well as to ignore uncertainty's negative side with the use of options: in its generic form, the right (the ability) but not the obligation (the need) to transact (to change).

Financial markets have learned how to manage both the positive and negative sides of uncertainty by using a plethora of hedging instruments, such as futures and options (puts and calls). The ability to manage uncertainty is a derivative of the capacity to hedge risks and can be found in its general form by the introduction of flexibility into project designs (Ramírez 2002).

A financial option is defined as the right to purchase or sell the underlying asset at a specific price with the right lasting for a specific period of time. This right can be interpreted as the ability to, or capability for, change (Ku 1995). It corresponds to the notion of actionable flexibility – the ability to respond to change by changing. Actionable flexibility is like buying an option to hedge a later course of action will be taken: the more flexible position chosen, the greater is the value of the option (Hirshleifer and Riley 1992). This relationship suggests that techniques based on option pricing theory can be used to assess flexibility within projects (real options).

"The capacity of uncertainty to be resolved in the future is precisely the characteristic that allows it to generate value" (Ramírez 2002, p.24). This represents the merger of uncertainty and risk, with real options, and the Bernanke concept of value in delaying decisions.

4.2.2 Corporate Finance and Capital Budget Decision-Making

Corporate finance deals with the monetary decisions that businesses make and includes the tools and analysis methods used to make these decisions. Its primary goal is to maximize overall value while managing risk, from the value of individual projects to corporate profits and ultimately shareholder value in the form of stock price. It includes long-term and short-term decisions and techniques.

4.2.2.1 Capital Budgeting Decisions

Capital investment decisions (budget decisions relating to the fixed assets and the capital structure of a business), are long-term choices about which projects receive investment, whether to finance that investment with debt or equity. These decisions also extend to the decision to pay dividends to shareholders. Conversely, short-term decisions deal with the short-term balance of current assets and current liabilities. They focus on the here and now operation of the project or entity directly *apropos* to inherent risk and the decisions to consume float. However, only the latter are germane to construction projects governed by network schedule systems. The long-term decisions will be disregarded hereinafter.

Capital budgeting is vital in decision-making. Managers must allocate limited resources between competing opportunities (projects). Making these decisions, formally known as capital allocations, requires estimating the value of each opportunity or project. This is a function of the size, timing and predictability of future cash flows compared to the uncertainty and risk of the business environment. It is the process of determining whether the value of the opportunity or project exceeds the cost of implementation.

4.2.2.2 Capital Budgeting Valuation Methods

Several formal methods are used in capital budgeting decisions. They rely on the measure of cash flows into and out of the firm or project. These methods fit into two categories (Ross et al. 2005): discounted cash flow methods and non-discounted cash flow methods. The primary difference being the Time Value of Money principle, i.e. a dollar today is not the same as, nor can it be compared to, a dollar in the future. These methods include techniques such as (Laudon and Laudon 2011):

Discounted Cash Flow Methods

- Net Present Value
- Cost-benefit Ratio
- Profitability Index
- Internal Rate of Return
- Modified Internal Rate of Return

Non-Discounted Cash Flow Methods

- Payback Period Method
- Return on Investment
- Accounting Rate of Return on Investment

Net Present Value (NPV): The difference between the present value of cash inflows and the present value of cash outflows minus the initial investment. NPV is an indicator of how much value an investment or project adds.

$$NPV = \sum_{t=1}^{T} \frac{C_t}{(1+r)^t} - C_0 \qquad \text{[Eq. 4.1]}$$

where

T = total project time

C_t = net cash flow at time t

C_0 = net cash flow at time $t = 0$ (the initial investment)

t = cash flow time

r = discount rate

The discount rate is the rate of return that could be earned on an investment in the financial markets with similar risk. It is also known as the opportunity cost of capital.

Table 4.1: Net Present Value (NPV) Decision Matrix

Value	Action
NPV > 0	Proceed with Project
NPV < 0	Do Not Proceed with the Project
NPV = 0	Indifferent: Evaluate Potential Strategic Value of Project

Cost Benefit Ratio (CBR): Also known as a cost-benefit analysis or conversely a benefit-cost analysis. It is an economic decision-making approach, used particularly by governments, to assess whether a proposed project, program or policy is worth doing or to choose between several alternatives. It involves comparing the total expected costs of each option against the total expected benefits to see whether the benefits outweigh the costs and by how much. Sometimes, a minimum cost-benefit ratio must be attained by capital projects to be accepted. Benefits and costs are expressed in terms of money and are adjusted for the time value of money using various discounted cash flow techniques.

$$\text{CBR} = \frac{\text{Total Benefits}}{\text{Total Costs}} \qquad \text{[Eq. 4.2]}$$

where

Total Benefits = the present value of the totality of benefits
Total Costs = the present value of the totality of costs

and

Net Benefit = Total Benefits – Total Costs

Profitability Index (PI): Also known as profit investment ratio (PIR) and value investment ratio (VIR). It is the ratio of payoff to investment (present value of benefits divided by the present value of costs) of a proposed project and is a useful tool for ranking projects, because it allows one to quantify the amount of value created per unit of investment. It requires that all future cash inflows (the net cash flow from each period) be adjusted for the time value of money.

$$PI = \frac{\text{Present Value of Cash Inflows}}{\text{Initial Investment}} \qquad \text{[Eq. 4.3]}$$

A PI of one indicates a breakeven endeavor. A PI lower than one indicates a present value that is less than the initial investment. As the value of the PI increases, so does the financial attractiveness of the proposed project.

Table 4.2: Profitability Index (PI) Decision Matrix

Value	Action
PI > 1	Proceed with Project
PI < 1	Do Not Proceed with the Project
PI = 1	Breakeven: Evaluate Potential Strategic Value of Project

Internal Rate of Return (IRR): Also known as the discounted cash flow rate of return (DCFROR) or the rate of return (ROR). It is used to measure and compare the profitability of investment in projects. It is the annualized effective compounded return rate that makes the net present value (NPV) of project (the sum of all cash flows, both positive and negative) equal to zero. More simply, the IRR is the discount rate at which the net present value of costs (negative cash flows) equals the net present value of the benefits (positive cash flows).

$$\text{IRR} = r, \text{ when NPV} = \sum_{t=1}^{T} \frac{C_t}{(1+r)^t} - C_0 = 0 \qquad \text{[Eq. 4.4]}$$

where

T = total project time

C_t = net cash flow at time t

C_0 = net cash flow at time $t = 0$ (the initial investment)

t = cash flow time

r = discount rate

If the IRR is greater than the cost of capital, accept the project. If the IRR is less than the cost of capital, reject the project.

Table 4.3: Internal Rate of Return (IRR) Decision Matrix

Value	Action
IRR > r_f	Proceed with Project
IRR < r_f	Do Not Proceed with the Project
IRR = r_f	Breakeven: Evaluate Potential Strategic Value of Project

r_f = financing rate (external cost of capital)

Modified Internal Rate of Return (MIRR): A modification of the internal rate of return (IRR) technique to resolve some IRR shortcomings: the single financing and reinvestment rate, and multiple IRRs for projects with alternating positive and negative cash flows. MIRR sums the negative cash flows after discounting them to time zero using the external cost of capital (r_f), sums the positive cash flows including the proceeds of reinvestment at the external reinvestment rate (r_i) to the final period, and then calculates rate of return that would cause the magnitude of the discounted negative cash flows at time zero to be equivalent to the future value of the positive cash flows at the final time period.

$$\text{MIRR} = \sqrt[T]{\frac{\text{FV(Positive Cash Flows, Using } r_i)}{-\text{ PV(Negative Cash Flows, Using } r_f)}} - 1 \qquad \text{[Eq. 4.5]}$$

where

FV = future value of cash flows at end of last period

PV = present value of cash flows at beginning of first period

T = total project time (number of equal periods at which cash flows end)

r_i = reinvestment rate

r_f = financing rate (external cost of capital)

As with unmodified IRR, a project is attractive when MIRR exceeds the project's hurdle rate. The hurdle rate, also referred to as the minimum attractive rate of return (MARR), is the minimum rate of return on a project that is acceptable before starting a project, given its risk and the opportunity cost of forgoing other projects. It is often decomposed into the sum of following components (Lang and Merino (1993).

- Traditional inflation-free rate of interest for risk-free loans
- Expected rate of inflation
- The anticipated change in the rate of inflation, if any, over the life of the investment (usually assumed at 0%).
- The risk of defaulting on a loan.
- The risk profile of a particular venture.

Payback Period Method: The period of time required for the return on an investment to repay the sum of the original investment. The time value of money is not taken into account. The payback period intuitively measures how long something takes to pay for itself. All else being equal, shorter payback periods are preferable to longer payback periods. It is widely used because of its ease of calculation but remains sensitive to odd cash flows.

$$\text{Payback Period} = \frac{\text{Cost of Project}}{\text{Annual Cash Inflows}} \quad \text{[Eq. 4.6]}$$

Return on Investment (ROI): A performance measure used to evaluate the efficiency of an investment. It is a percentage represented by the total revenue at the end of the project period less the initial investment divided by the initial outlay of the project.

$$\text{ROI} = \frac{\text{Gain from Investment} - \text{Cost of Investment}}{\text{Cost of Investment}} \quad \text{[Eq. 4.7]}$$

If an opportunity does not have a positive ROI, or if there are other opportunities with a higher ROI, then it should be not be undertaken.

Accounting Rate of Return on Investment (ARR): Also known as the average rate of return. It is the return generated from the net income of the proposed capital investment and is a percentage. ARR does not take into account the time value of money.

$$\text{ARR} = \frac{\text{Average Profit}}{\text{Average Investment}} \quad \text{[Eq. 4.8]}$$

where

$$\text{Average Investment} = \frac{\text{Book Value at Beginning} + \text{Book Value at End of Useful Life}}{2} \quad \text{[Eq. 4.9]}$$

If the ARR is equal to or greater than the required rate of return, accept the project. If it is less than the desired rate, reject the project. When comparing opportunities, the higher the ARR, the more attractive the opportunity.

Table 4.4: Accounting Rate of Return (ARR) Decision Matrix

Value	Action
ARR > *r*	Proceed with Project
ARR < *r*	Do Not Proceed with the Project
ARR = *r*	Breakeven: Evaluate Potential Strategic Value of Project

A capital budgeting analysis conducted using one or more of the aforementioned methods tests to see if the benefits (i.e. cash inflows, also known as profits) are large enough to overcome three things: (1) the cost of the asset, (2) the cost of financing the asset (e.g. interest), and (3) the desired rate of return (the risk premium) that compensates for potential errors made when estimating cash flows in the future.

When faced with choosing among alternative projects, managers use different and/or multiple techniques to determine which project to undertake. Each technique has its advantages and disadvantages, focusing on a specific aspect of the decision. In a world of complete certainty, net present value is the most suitable valuation technique. However, since the world is not at all certain and involves nonlinearities, more advanced techniques are necessary for proper project evaluation (Quispez-Asin 2008).

4.2.2.3 Challenges with Non-Discounted Cash Flow Methods

Capital budgeting methods are challenged by inherent and subjective limitations in their application. One category, non-discounted cash flow methods, ignores the time value of money altogether, while the majority of those recognizing it foster uniformity with subjectivity in the application of discount / interest rates.

The payback period method is insensitive to the time value of money and does not consider the value gained beyond the payback period, thereby demonstrating virtual blindness to the entire stream of future cash flows. Although it has disadvantages, the method has appeal due to its simplicity and underlying conservative-minded assumption that long-term forecasts of cash flows are inaccurate (de Neufville 2009).

The return on investment is appealing due to its ease of calculation and understanding. Much like the payback period, ROI ignores the time value of money but does take into consideration the cash flows over the lifetime of the project. Therefore, it is also unsuitable to value a project correctly (de Neufville 2009).

The accounting rate of return on investment method does not consider the time value of money. It has inherent flaws much like the payback period method. In general, managers and investors are focused on cash inflows from the project into their organization. As ARR is based on numbers that include non-cash items, this singular focus becomes muddied. Also, an important difference with an ARR valuation versus payback period is that ARR tends to favor higher risk decisions, because future profits are insufficiently discounted for risk, as well as for the time value of money. Conversely, the payback period method leads to overly conservative decisions.

4.2.2.4 Challenges with Discounted Cash Flow Methods

When considering methods that discount cash flows for the time value of money, the most ubiquitous method is net present value. Of particular importance to an NPV calculation is the rate at which cash flows are discounted to take into account the time value of money. The weighted average cost of capital (WACC) is the opportunity cost of the money being used to finance the company (the combined cost of money from equity and debt). WACC is used as the discount rate in NPV calculations; the rate applied and its origin remain as subjective elements. The discount rate applied in an NPV calculation reflects the risk adjusted rate of return that could be earned elsewhere by investing in projects of similar risk (Quispez-Asin 2008).

Another challenge NPV valuations present is that the actual rate at which cash inflows materialize can be very uncertain, and the assumption that cash flows are completely predictable is severely flawed. Thus, the straightforward NPV method must be supplemented with the recognition and measure of uncertainty that does not simply look at the most likely scenarios (Spinler 2007).

When considering discount rates other than the WACC in NPV or other discounted cash flow methods, low discount rates favor capital intensive projects with strong positive cash flows anticipated to materialize in the distant future. Conversely, high discount rates favor projects whose cash inflows will be received relatively soon.

The cost-benefit analysis represents another method that includes subjectivity and elements beyond monetary cash flows. The accuracy of the outcome of a cost-benefit analysis depends on how accurately costs and benefits have been estimated (Flyvbjerg et al. 2002). Studies indicate that the outcomes of cost-benefit analyses should be treated with caution, because they may be highly inaccurate and lead to inefficient decisions.

These outcomes (almost always tending towards underestimation) are to be expected, because such estimates (Flyvbjerg et al. 2005):

1. Rely heavily on similar past projects (often differing markedly in function or size and certainly in the skill levels of the team members).
2. Rely heavily on the project members to identify (remember) the significant cost drivers from projects in their collective past.
3. Rely on very crude heuristics to estimate the monetary value of the intangible elements.
4. Are unable to completely dispel the biases of the team members (who often have a vested interest in a positive decision) and their natural tendency to think and only present the positive.

Beyond this, another challenge to cost-benefit analyses arises with the determination of which costs should be included in the analysis, the significant cost drivers. This derivation is often controversial as competing interests may put forward differing costs for inclusion or exclusion in the valuation calculation.

Often confused with an NPV calculation, the profitability index is a hybrid of NPV and is similar to the internal rate of return calculation, in that it may give the wrong decision when choosing between mutually exclusive projects. PI calculations for such projects, for which only one will be accepted (the most profitable one), do not differentiate between project durations. This leads to the tendency for projects with longer durations being portrayed as better that those with shorter durations, despite discounting cash flows. Where the profitability index calculates the 'relative value of the investment' with respect to the expected return, the net present value calculates the 'actual value of the investment.'

The primary difference in these calculations is that PI does not take the initial investment amount into consideration and should be excluded from use where investment amount is limited.

Directly related to NPV, the internal rate of return technique is the most commonly used method for evaluating capital budgeting proposals. However, IRR is flawed in that its calculation may not always result in a unique solution. That is, there can be multiple rates of return for the same project depending on the order of cash flows particularly for projects with alternating positive and negative cash flows. The IRR does not always correlate with the NPV, meaning that an increasing NPV does not necessarily translate into a decreasing IRR (de Neufville 1990). Like NPV, IRR suffers from the flaw of averages, as the most likely cash flows are used to determine its value. In practice, the IRR cannot be determined directly using a formula; it must be approached on a trial-and-error basis.

While there are several problems with the IRR, the modified internal rate of return resolves two. First, IRR assumes that interim positive cash flows are reinvested at the same rate of return as that of the project that generated them (Kellerher and MacCormack 2004). Usually, funds will be reinvested at a rate closer to the firm's cost of capital. Second, more than one IRR can be found, which leads to confusion and ambiguity. The MIRR results in only one value.

Beyond the inherent shortcomings and subjectivity, traditional capital budgeting methods have difficulty in valuing project abandonment, deferral, or alteration in any way (Mbuthia 2001). They cannot adequately recognize managerial operating flexibility and strategic interactions. The end result is an “inability to properly recognize the value of active management in adapting to changing market conditions" (Trigeorgis 1996, p.9). Traditional valuation techniques are also prone to managerial bias, especially in the selection of discount rates.

4.2.2.5 The Flaw of Averages

Common to most capital budgeting valuation techniques is that their calculations are based on only the most likely or average scenarios of project performance. This can have disastrous consequences, as posited by Savage in the *The Flaw of Averages*, which states: "Plans based on the assumption that average conditions will occur are usually wrong" (Savage 2000, p.1). Applicable insight to valuation techniques is that when managers are faced with uncertain future states of a project, they tend to make decisions and plans based solely on average conditions. This presents a problem, should there be wide variation in periodic results. When projecting cash flows, large initial outflows coupled with anticipated large inflows in later years result in an average not reflective of the capacity or needs of the project being evaluated.

An example demonstrating the shortcomings put forward by *The Flaw of Averages* surrounds the capacity of a hypothetical computer distribution center where the average modeled throughput is 1,000 computers per month. Periods of high demand can reach 1,500 units per month. When this occurs, a system design based solely on average throughput will not be able to accommodate the additional computers (Quispez-Asin 2008). Conversely, should the monthly demand reach only 500 computers, the cash inflows may not be sufficient to sustain operations.

From a more rigorous mathematical perspective, Jensen's inequality (Eq. 4.10) states that the expected value of a function whose input is a random variable is not necessarily equal to the function of the expected value of the random variable, especially when the functions are nonlinear (de Neufville 1990). Accordingly, NPV calculations based solely on the expected values of inputs cannot satisfactorily provide a sufficient projection for a capital budgeting decision.

$$E[g(x)] \leq g(E[x]) \qquad \text{[Eq. 4.10]}$$

4.2.2.6 Biases in the Application of Capital Budgeting Techniques

Biases, the preconceived notions, foregone conclusions, predispositions and/or tendencies known or unknown, make their way into traditional methods of analyzing capital budgeting projects. Some methods have a tendency to make one project's performance appear better than the others, e.g. small vs. large projects, short-lived vs. long-lived projects.

For example, consider two proposed projects. Project A has an initial investment of $100,000 and generates average cash inflows of $23,250 for five years; and Project B has a $300,000 initial investment and $37,500 average cash inflows for ten years. Discount rates remain constant at four percent and a reinvestment rate where applicable at three and one-half percent. Results from several of the aforementioned capital budgeting decision-making techniques are summarized in Table 4.5. It depicts the biases such as the scale effect and unequal lives, when considering mutually exclusive opportunities.

Table 4.5: Capital Budgeting Technique Bias Calculations

Project	Initial Cost	Average Cash Inflows	Average Annual Rate of Return	Net Present Value (NPV)	Profitab-ility Index (PI)	Modified Internal Rate of Return (MIRR)	Payback Period	Return on Investment (ROI)
A	$100k	$23,250 (5-years)	23.25%	$3,370	1.035	4.51%	4.30	16.25%
B	$300k	$37,500 (10-years)	12.50%	$4,000	1.014	3.90%	8.00	25.00%
Favors	A	B	A	B	A	A	A	B

When looking solely at the most frequently used decision/valuation technique, net present value, Project B is favored over Project A given the higher value: NPV of $4,000 vs. $3,370 respectively. When considering the per dollar invested return (the profitability index technique,) Project A has the higher return, a clear example of the scale bias of NPV. Similarly, when basing a decision on initial investment, modified internal rate of return, and payback period, Project A is favored. Average cash inflows and return on investment support Project B.

In Table 4.5, Projects A and B have different durations – five years vs. ten years. Again, when deciding between mutually exclusive projects, a bias favors longer-lived projects. Everything else remaining the same, a project with the longer life will demonstrate a higher NPV, as is the case with Project B, despite returning lesser average annual rates of return, i.e. profitability (as demonstrated through the PI) and requiring a longer period to payback the initial investment.

When considering capital investments for equipment, the same bias for longer-lived assets exists. Considering an investment in a machine with an expected life of three years to one with a five-year life, the five-year asset will have a higher NPV and represent a better investment. It has to do with the amount of interest earned on the reinvestment of the cash inflows over a longer period of time. As discussed with the IRR shortcomings versus MIRR, the NPV method uses the time value of money principle to arrive at an answer. Built into NPV calculations is the assumption that:

- The cash flows will be reinvested to earn a rate of return (interest) for the remainder of the project's life.
- The rate that is assumed to be earned is the discount rate used to determine the present value factors.

Bias evaluation and compensation are necessary to avoid making an incorrect decision. One characterized as a type II error (from statistical analysis and social science flaws in logic, reasoning, and hypothesis testing) in this case, accepting a project that will lose money, as opposed to a type I error of not accepting a project that will be profitable. Most consider the type II error the more serious, as it leads to an actual, realized loss, as opposed to no more than a lost opportunity.

4.2.3 Financial Options Theory

In its broadest sense, an option is defined as "…a security giving the right to buy or sell an asset, subject to certain conditions, within a specific period of time" (Black and Scholes 1973, p.637). According to Hull (2000), stock options were first traded on an organized exchange in 1973. In the financial arena, an option is an instrument that establishes a contract to buy or sell a security (typically a stock, a bond, currency, or a commodity) at an agreed upon price during a specified period, or on a specific date in the future, and is commonplace in contemporary financial markets and portfolio theory.

4.2.3.1 Financial Options Defined

In the financial realm, options are derivative financial instruments based on an underlying asset (generally stocks, bonds, currency, commodity futures, etc.) that form a contract between parties to buy (a call option) or sell (a put option) the underlying asset at a predetermined fixed price (the exercise price or strike price) on or before a specified expiration or exercise date (Black and Sholes 1973). The important feature of an option, either a put or call, is that its purchase / ownership conveys the right to complete the transaction should it be advantageous but not the obligation to do so when not financially advantageous. American-style options give the holder the right to complete the transaction or 'exercise the option' anytime before its expiration, while European-style options can only be executed on the expiration date. Contained within Table 4.6 are terms and concepts relative to options theory and practice.

The price paid for an option, its premium, differs from its value. Its value is the ultimate profit returned to its owner should the transaction be completed. Its price is derived from the difference between the exercise price and the current value of the asset in addition to a premium based on the time remaining until the expiration of the option. The value of an option to its owner (its purchaser or holder) is theoretically unlimited. Conversely, the financial risk to the purchase of an option is limited to the premium, as by definition the owner of an option is not obligated to complete a financially detrimental transaction. The premium is simply lost.

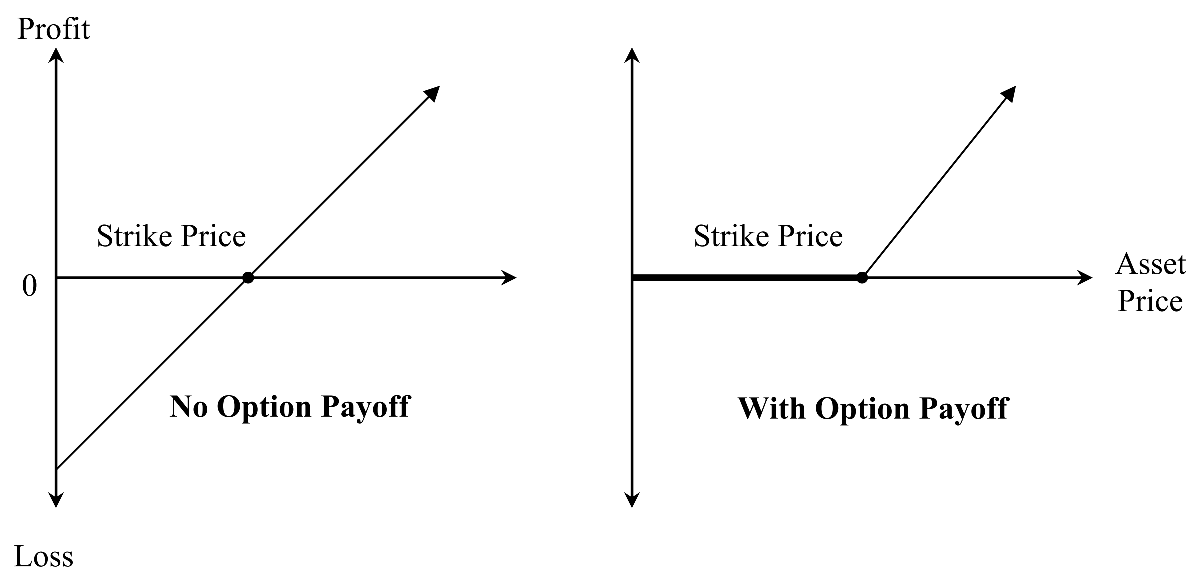
Table 4.6: Option Terminology (adapted from American Stock Exchange *et al*. 1994)

Term	Meaning
Option	Right to either buy or sell a specified amount or value of a particular underlying asset at a fixed price on or before its expiration
Option Holder	Owner of the option, also referred to as the option buyer
Option Writer	Obligated to perform according to the terms of the option, also referred to as the option seller
Premium	Price the holder of an option pays the writer of the option; it is not a down payment or partial payment and is entirely non-refundable
Call Option	Option giving the holder the right to buy the underlying asset at a predetermined price
Put Option	Option giving the holder the right to sell the underlying asset at a predetermined price
Exercise	Decision to move forward to buy or sell the underlying asset in accordance with the terms of the option contract
Exercise Price	Price at which the option holder has the right to either purchase or sell the underlying asset, also referred to as the strike price
Expiration Date	Date on which the option expires, assuming it has not been exercised prior to the expiration date, in which case it ceases to exist
At-the-Money	When the current market value of the underlying asset is the same as the exercise price of the option
In-the-Money (Call Option)	When the current market value of the underlying asset is above the exercise price of the option
In-the-Money (Put Option)	When the current market value of the underlying asset is below the exercise price of the option
Out-of-the-Money (Call Option)	When the exercise price of the option is above the current market value of the underlying asset
Out-of-the-Money (Put Option)	When the exercise price of the option is below the current market value of the underlying asset
Long	Position as the holder of an option
Short	Position as the writer of an option
Style of Option	Defines when the option is exercisable, e.g. American vs. European
Underlying Asset	A financial instrument in the form of a stock, a bond, currency, a commodity, etc.

4.2.3.2 Option Payoffs

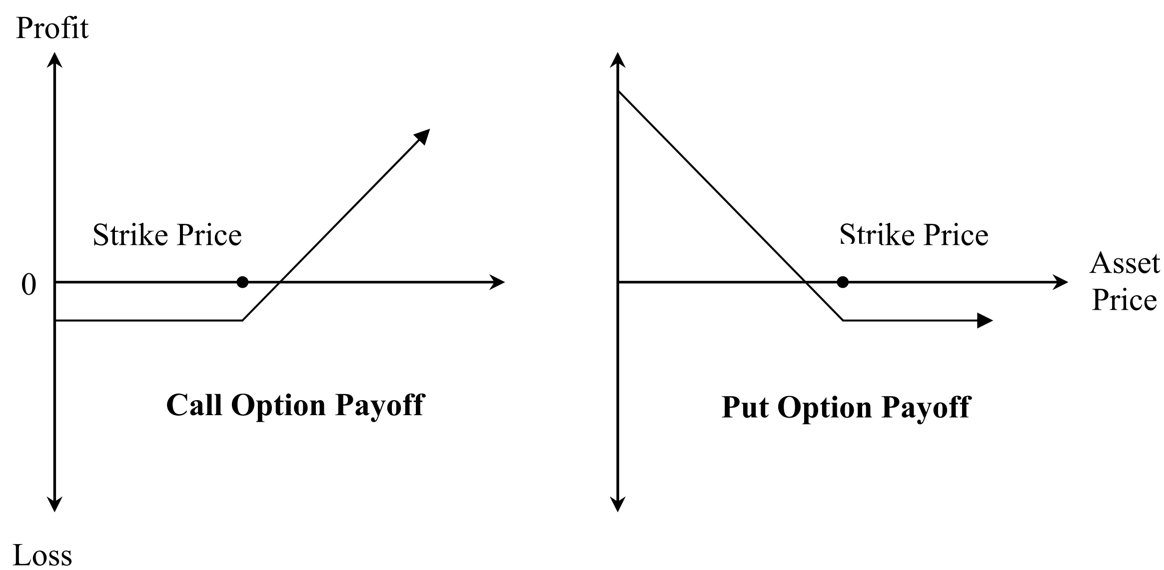
The benefit to the owner of an option is an increase in the upside potential of the investment should the underlying option actually be bought or sold, or an in-the-money option (an option whose underlying asset value has changed such that the premium has been recovered) that can be sold to another investor.

Figure 4.1: Payoffs from Option Ownership



Call Options: The owner of a call option has the expectation that the price of the underlying asset will rise in the future. If the asset price at the exercise date is above the strike price by more than the premium paid, a profit is realized. Call option profits for the buyer can be very large and are limited only by the extent to which the underlying asset price increases. Conversely, the seller of a call option does not expect the price to rise or is willing to give up some of the upside (profit) for the premium (an immediate payment) and retain the opportunity to make a gain up to the strike price. If the asset price at expiration date is lower than the exercise price, the owner will let the call contract expire (as it is worthless) and only lose the amount of the premium paid for the option. Hence, the risk downside risk for the owner of a call option is limited to the premium.

Figure 4.2: Payoffs from Put vs. Call Option Ownership



Put Options: The owner of a put option has the expectation that the price of the underlying asset will decrease in the future. If the asset price at the exercise date is below the strike price by more than the premium paid, a profit is realized. Put option profits for the buyer are limited by the price of the underlying asset at the time the option is purchased. Conversely, the seller of a put option does not expect the price to fall. If the asset price at expiration date is higher than the exercise price, the owner will let the put contract expire (as it is worthless), and only lose the amount of the premium paid for the option. Likewise, the risk downside risk for the owner of a put option is limited to the premium.

Negative Payoffs: The primary purpose of financial options is to provide the owner the opportunity to wait for time to elapse and information/events to materialize that have the ability to impact the price of the asset and then decide whether to buy or sell it. This provides downside protection against losses, such that there are no negative payoffs (Trigeorgis 1993) beyond the sunk cost of the initial option premium.

4.2.3.3 Option Pricing – The Black-Scholes Method

The value of an option, financial or real, is determined by one of several mathematical models. These models attempt to predict how the value of an option changes in response to the changing conditions in which they exist. Nested within the valuation method are five key considerations (Frayer and Uludere 2001): the value of the asset being optioned, the exercise or strike price, the time to expiration (or to the evaluation / decision point), volatility of the asset in its market or setting, and the risk-free interest rate.

Originally, option pricing theory was limited to European-style options. It was first articulated in the paper *The Pricing of Options and Corporate Liabilities* (Black and Scholes 1973), in which the authors derived a partial differential equation (Eq. 4.11) that defines the price of options over time (as in the case of European-style options at the expiration date).

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0 \qquad \text{[Eq. 4.11]}$$

where

S = price of the stock (the underlying asset)
V = price function of an option at payoff, a function of time (t) and stock price (S)
r = annualized risk-free interest rate, continuously compounded.
σ = volatility of the stock's returns
t = time in years

The seminal breakthrough was to perfectly hedge the option, to take an investment position intended to offset potential losses that may be incurred by a companion investment, buying and selling the underlying asset in just the right way and consequently eliminating the risk. Ultimately, Robert Merton, the first to publish a paper expanding the mathematical understanding of the model, and Myron Scholes were awarded the 1997 Nobel Prize in Economics, after the 1995 death of Fischer Black.

The value of a European-style call option can be determined using the Black-Scholes Equation as follows (Black and Scholes 1973, p. 647):

$$C(S,t) = N(d_1)S - N(d_2)Ke^{-rT} \qquad \text{[Eq. 4.12]}$$

where

N(y) = probability, the standard normal distribution
K = strike price of the option
T = time to maturity
S = current stock price (the underlying asset)
r = compounded annual risk free rate
σ = volatility of the returns of the underlying asset

and,

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \qquad \text{[Eq. 4.12a]}$$

$$d_2 = d_1 - \sigma^2\sqrt{T} \qquad \text{[Eq. 4.12b]}$$

The price of a call option involves the acquisition of stock equal to the $N(d_1)S$ portion of Eq. 4.12 – where this term is equivalent to the expected value of the stock when $S > K$ (when the current stock price is greater than the strike price) using neutral probabilities, and then borrowing against the stock (at interest rate *r*) in an amount equivalent to $N(d_2)Ke^{-rT}$ – where Ke^{-rT} represents the present value if the cost of the option and $N(d_2)$ is the risk neutral probability of S > K at the expiration date (Amram and Kulathilaka 1999). The equation is broken down into its component parts in Figure 4.3.

Figure 4.3: Analytic Breakdown of the Black-Scholes Equation (modeled after Amram and Kulathilaka 1999)

Risk neutral probability of asset at expiration

$$C(S,t) = N(d_1)S - N(d_2) \bullet Ke^{-rT}$$

Expected value of asset based on risk neutral probabilities

Present value of cost of investment

This is all predicated upon the assumption that the volatility of the stock (its change in price over the option period), follows Geometric Brownian Motion (GBM), also known as the Wiener process (Hull 2005). In the Eq. 4.13, μdt represents the deterministic change in value of the underlying asset (a function of the instantaneous growth rate in the stock price over time μ) and σdW represents the stochastic change in stock price (a function of the standard deviation of the GBM/Weiner process) and is normally distributed with a mean of zero and increases with the interval *dt* (Menassa 2009).

$$\frac{dS}{S} = \mu dt + \sigma dW \qquad \text{[Eq. 4.13]}$$

Despite its apparent complexity, the Black-Scholes Equation is an easy method to value an option. It requires five inputs to make the computation. Table 4.7 differentiates between the five needed inputs and the surrounding information considered superfluous.

Table 4.7: Information Needed to Price and Option (adapted from Amram and Kulathilaka 1999)

Information Needed to Price an Option	Superfluous Information
Current Asset Value: The current value of the underlying asset, which is observed in the market	*Future Asset Value:* Probability estimates are not needed, because they are captured by the current value of the underlying asset and the volatility estimate
Exercise Date: The time to the decision date, which is defined by the features of the investment	*Discount Rate:* An adjustment to the discount rate for risk is not needed, because the valuation solution is independent of anyone's taste for risk
Strike Price: The investment cost or exercise price (strike price), which is defined by the features of the investment	*Expected Profit (Option):* The expected rate of return of the option is not needed, because the option is valued directly by dynamic tracking
Risk-free Interest Rate: The risk-free rate of interest, that is observed in the market *Asset Volatility:* The volatility of the underlying asset, which is often the only estimate input. Cash payouts or non-capital gains returns to holding the underlying asset, which are often directly observed in the market, or sometimes estimated from related market.	*Expected Profit (Asset):* The expected rate of return for the underlying asset is not needed, because the value of the underlying asset and the ability to form tracking portfolios already capture its risk/return tradeoff.

The Black-Scholes Model remains one of the most important methods and foundations for the existing financial market in which the result is within the reasonable range (Hull 2005).

4.2.3.4 Option Pricing – Finite Difference Method

The Black Scholes Equation fits within the category of finite difference methods, an expression of a partial differential equation. Finite difference methods solve option pricing problems when other methods are inappropriate or difficult to implement, and are considered limited in terms of the number of underlying variables that may be included. First applied in

1977 by Schwartz, an option is valued using finite differences based upon the following concepts and process (Wilmott et al. 1995):

- Maturity values are the difference between the exercise price of the option and the value of the underlying asset at each point in time.
- Values at the boundary prices are set based on arbitrage bounds on option prices.
- Values at other points (lattice points in similar form to decision trees) are calculated recursively (techniques such as Crank-Nicolson or an explicit method may be used):
    1. The partial differential equation is discretized per a suitable mathematical technique, such that the value at each point is specified as a function of the value at later and adjacent points – similar to numerical analysis.
    2. The value at each point is found using the selected mathematical technique.
- The present value of the option, where the underlying asset is at its spot price, is determined by interpolation.

4.2.3.5 Option Pricing – Monte Carlo Simulation

Another approach to valuing options is the application of Monte Carlo simulation. It has the potential to overcome the intractable nature of complex partial differential equations, particularly when applied to American-style options. A Monte Carlo approach to option valuation produces thousands of possible scenarios for the uncertain elements to generate random price paths of the underlying asset, each of which results in a payoff for the option. The option price obtained from a Monte Carlo simulation is a sample average (Chance 2011) for which a payoff value can be discounted to yield an expectation value for the option.

To arrive at the price of the option the Monte Carlo technique follows a multi-step process: (1) generate several thousand possible (but random) price paths for the underlying asset, (2) calculate each associated payoff of the option – one per simulation iteration,

(3) determine the average payoff value, and (4) calculate the present value of the average payoff value (Crack 2004).

One of the strengths of simulation modeling is that it allows specific path-dependent options to be valued. Monte Carlo simulation should give the same result as the more rigorous Black-Scholes and the binomial valuation methods, particularly when based on the risk neutral probabilities (Masunaga 2009).

4.2.3.6 Option Pricing – Binomial Valuation Method

Nested within decision analysis is the ability to structure the problem at hand, its uncertainties and contingent decisions explicitly in the form a decision tree. A decision tree is a series or sequence of decisions at discrete points of time represented by nodes that ultimately reach a group of terminal nodes. Terminal nodes represent the point at which the decision-maker faces a final decision. Interim nodes and the branches they form represent the variety of choices along the path to a terminal node. All decisions in a tree structure must be mutually independent.

Decision tree analysis recognizes that only through the resolution of uncertainty is the appropriate decision reached at each point in time. There is no pre-commitment to a particular decision at any point in time. Rather, decision tree analysis identifies an array of decisions, each of which can be considered as optimal under the variety of uncertainties and evolution of the project over time (de Neufville 1990).

The Binomial Option Pricing Method, an extension of decision tree analysis, developed by Cox et al. (1979), values options to buy or sell financial assets at defined time intervals. It depicts two possible changes in value for an asset in each time period represented as a decision node (a move up or a move down) and greatly simplifies valuation calculations. It eliminates the need to estimate the premium for risk in the discount rate. Combining a binomial decision approach with the decision tree approach allows the model to illustrate the intermediate decision-making processes, from beginning to option expiration. The Binomial Method promotes intuitive understanding of the decisions at each point in time (Masunaga 2009).

Valuation under the binomial method is an iterative process. It begins at each of the final nodes and works backwards through the tree to the first node to arrive at an initial value for the option. A value is computed at each stage that represents the value of the option at the given point in time. Valuation using this method is a three-step process (Cox et al. 1979): (1) Generate the option price decision tree, (2) calculate the option value at each final node, and (3) sequentially calculate the option value at each preceding decision node.

Depending on the style and type of the option being priced, continuation of the valuation process requires determination of the possibility of early exercise at each node based upon the following criteria: can the option can be exercised, and if so, does the exercise value exceeds the binomial value. If the value exceeds the binomial value, the option at the node being considered is in-the-money. Exercise the option based on the following considerations:

- European-style Option: Since there is no opportunity for early exercise by definition, the binomial value applies at all nodes.
- American-style Option: Since the option may be held or exercised any time prior to its expiration date, the value at each node is the greater of the Binomial or the exercise value.

- Call Option vs. Put Option: At each node, option value is determined by:

  Call Option: The maximum of $(S_n - K)$, or zero

  Put Option: The maximum of $(K - S_n)$, or zero

  where

  K = strike price of the option
  S_n = current stock price (the underlying asset) at the n^{th} period

This value can approximate the theoretical value produced by the Black-Scholes Equation to the desired degree of precision. However, the binomial model is considered more accurate than Black-Scholes, because it is more flexible. Discrete future dividend payments can be modeled correctly at the proper forward time steps, and American-style options can be modeled as well as European-style options.

4.2.3.7 Binomial Decision Trees vs. Decision Lattices

When comparing a to a decision lattice (Figure 4.4) to decision tree (Figure 4.5), a seminal difference appears. Decision lattices are recombining while decision trees are not. That is, at each node in a decision tree, two distinct decisions exist: up or down with probabilities unique to each decision that result in unique paths forward. This results in exponential growth of terminal nodes, representing a vast array of option values to consider.

As for a decision lattice, while the possible outcomes at each node remain the same as up or down, it does not produce two unique subsequent decision nodes. Rather, the inner nodes recombine the adjacent up and down decisions to a singular node, reducing the number of subsequent nodes such that each period only increases by one node.

Figure 4.4 Decision Lattice
(modeled after Brandão et al. 2005*)

* Adapted with permission from Brandão, L., Dyer, J. and Hahn, W. Copyright (2005), INFORMS, Hanover, MD.

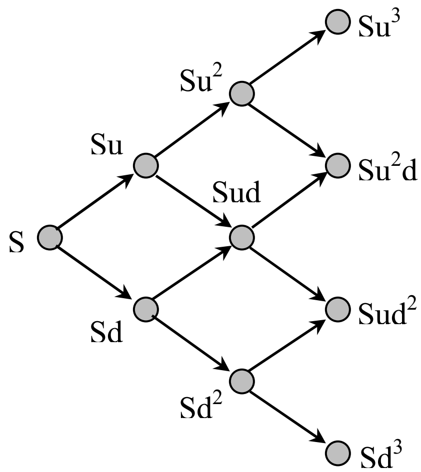
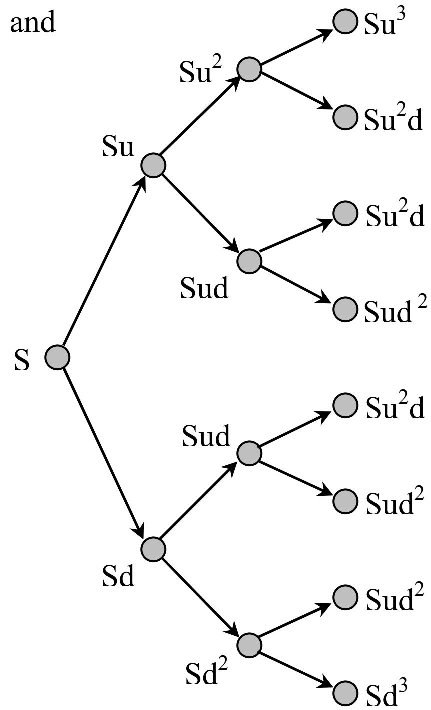


Figure 4.5: Decision Tree
(modeled after Brandão et al. 2005*)



where

S = current stock price (the underlying asset)
S_{u-d}= stock price at subsequent decision node
u = decision in the upward direction
d = decision in the downward direction

Analysis by decision trees is a powerful graphical depiction of the effects of uncertain events. It allows for flexibility in making real-time decisions (de Neufville, 1990),and the realization of the best decision under uncertain conditions. The strength in decision tree analysis is that it allows the consideration of the impact of entirely different uncertainties without limiting the growth rates of the predetermined evolution of probabilities (Chambers 2007). The accuracy of decision trees depends largely on the ability to correctly structure the probabilities and costs of future decisions, the decisions and outcomes represented at each node.

The strength of the binomial lattice is its ability to reduce a large set of potential outcomes (payoffs) to a manageable size (Chambers 2007), as depicted in Table 4.8. Within the context of financial options, the price of an asset, generally a stock, undergoes random variation in its value. Were it assumed that the stock price would only increase or decrease in value by a fixed amount during each period, the number of possible binomial decision tree outcomes grows exponentially by 2^n, representing the *n* periods. Should an investor desire to determine a stock price over a 20 day period (n=20), 2^{20} returns over one million possibilities (terminal nodes) to evaluate.

Table 4.8: Decision Tree vs. Lattice Terminal Node Quantity

Decision Period (*n*)	Decision Lattice Nodes	Decision Tree Nodes
0	1	1
1	2	2
2	3	4
3	4	8
18	19	262,144
19	20	524,288
20	21	1,048,576

Where decision tree analysis and the binomial lattice diverge surrounds path independence versus path dependence. Decision trees require path dependence. That is, an analysis using decision trees does not result in the same solution along each distinct path. Returning to the nomenclature of the generic decision tree depicted in Figure 4.5, the terminal node second from the top of the tree represents the following path-specific set of decision choices:

Su^2d (2^{nd} from top) = Su (Decision 1) • Su (Decision 2) • Sd (Decision 3) [Eq. 4.14]

Similarly, the fifth from the top terminal node also culminates in the same combination of up and down choices (two up, one down), but represents a uniquely different set of path-specific decision choices that are not considered equal. The opportunity for different probabilities for outcomes at each node can result in markedly different values for the option:

Su^2d (5^{th} from top) = Sd (Decision 1) • Su (Decision 2) • Su (Decision 3) [Eq. 4.15]

Conversely, the binomial decision lattice requires path independence. That is, irrespective of the specific sequence of decision choices, the terminal node value must be equal. Returning to the nomenclature of the generic binomial lattice depicted in Figure 4.4, the terminal node third from the top of the tree can be reached by three different sets of decision choices, each of which returns the same terminal value:

Sud^2 (Path 1) = Sd (Decision 1) • Sd (Decision 2) • Su (Decision 3) [Eq. 4.16]

Sud^2 (Path 2) = Sd (Decision 1) • Su (Decision 2) • Sd (Decision 3) [Eq. 4.17]

Sud^2 (Path 3) = Su (Decision 1) • Sd (Decision 2) • Sd (Decision 3) [Eq. 4.18]

where

Sud^2 (Path 1) [Eq. 4.15] = Sud^2 (Path 2) [Eq. 4.6] = Sud^2 (Path 3) [Eq. 4.17] [Eq. 4.19]

Use of the lattice method provides distinct advantages. It makes visible numerous uncertain possibilities with relative ease and permits mathematical expression of terminal probabilities using one variable. The probabilities for a three period binomial decision lattice are summarized in Table 4.9.

Table 4.9: Probability of Occurrence at Binomial Decision Lattice Nodes

Initial	Decision Period 1		Decision Period 2		Decision Period 3	
State	Formula	Value	Formula	Value	Formula	Value
S = 1	Su = p	0.80	$Su^2 = p^2$	0.64	$Su^3 = p^3$	0.512
	Sd = (1-p)	0.20	Sud = 2p(1-p)	0.32	$Su^2d = (p^2+2p^2)(1-p)$	0.384
			$Sd^2 = (1-p)^2$	0.04	$Sud^2 = 3(p-p^2)(1-p)$	0.096
					$Sd^3 = (1-p)^3$	0.008

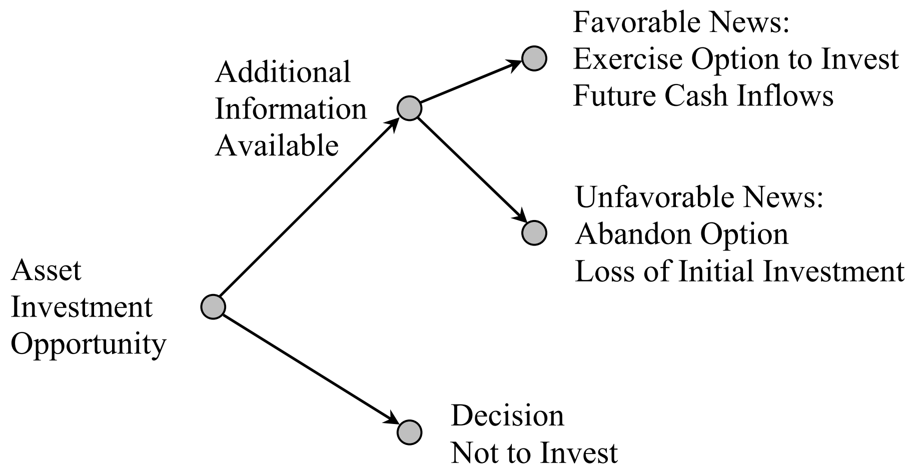
where
p = probability of up decision (values calculated at p = 0.80)
(1 – p) = probability of down decision

4.2.4 Real Options Theory

The term *real option* has become a catch-all phrase that encompasses the uncertainty surrounding strategic business decisions and the uncertainty surrounding their exaction. In fact, real options do not solve every decision-making problem, nor are they intended to (Janney and Dess 2004). Rather, they are a two-part decision tool with an initial opportunity to take beneficial action with an inherent wait-and-see feature and the subsequent opportunity to make a decision upon the revelation of new information.

Traditional decision analysis without the application of option theory assumes a now-or-never investment strategy in keeping with the Bernanke precept. It ignores the alternative opportunity to wait to invest at a later date, whereby the potential to increase profitability is found as new information becomes available. This is the application of option theory to real projects. The seminal concept with respect to real options is that the greater the potential outcome, the more valuable waiting to make a decision becomes (Janney and Dess 2004).

Figure 4.6: Real Option Decision Tree Structure



4.2.4.1 Real Options Defined

Real options are founded upon the same principles as financial options, but give the owner the right, but not the obligation, to take future action with respect to real, tangible assets, depending on how uncertain conditions evolve (Amram and Kulatilaka 1999). It is presupposed that when considering real options, real tangible assets are actually owned (Frayer and Uldere 2001).

The term *real options* was first used by Myers (1984) in the context of strategic corporate planning and capital budgeting. Myers recognized that the right, but not the obligation to take action, can help maintain or even increase the value of a project despite uncertainty. Commenting on capital budgeting and finance, Myers noted that capital budgeting and strategic planning emerged post World War II "as two cultures looking at the same problem" (Trigeorgis 1999, p.7) and concluded that:

> Strategic planning needs finance. Present value calculations are needed as a check on strategic analysis and vice versa. However, standard discounted cash flow techniques will tend to understate the option value attached to growing profitable lines of business. Corporate finance theory requires extension to deal with real options (Myers 1984, p.136).

Over time, the application of real options has broadened to include multiple types of decision-making in uncertain conditions. The underlying premise of real options is that wherever there is a choice, there is the ability to benefit from the upside, while avoiding or postponing any downside risk. When new information becomes available, when the uncertainty surrounding the conditions in which the asset resides or project operates is resolved, and when cash flows become more certain, managers may depart from their original strategy and revise the operational path forward (Dixit and Pindyck 1994). Using the analogous financial option, the flexibility provided managers to revise strategy is the definition of a real option (Boute et al. 2004).

Wang (2004) concludes that consensus regarding a definition of real options does not exist and presents a summary of the multiple definitions and understandings of real options:

- "In a narrow sense, the real options approach is the extension of financial option theory to options on real (nonfinancial) assets." (Amram and Kulatilaka 1999, p.7)
- "Similar to options on financial securities, real options involve discretionary decisions or rights, with no obligations, to acquire or exchange an asset for a specified alternative price." (Trigeorgis 1996, p.xi)
- "An opportunity is an option – the right but not obligation to take some action in the future." (Dixit and Pindyck, 1995, p.1)

- "A real option is the right, but not the obligation, to take an action (e.g. deferring, expanding, contracting, or abandoning) at a predetermined cost called the exercise price, for a predetermined period of time – the life of the option." (Copeland and Antikarov 2001, p.5)
- "In fact, it is possible to view almost any process that allows control as a process with a series of operational options. These operational options are often termed real options to emphasize that they involve real activities or real commodities, as opposed to purely financial commodities, as in the case, for instance, of stock options." (Luenberger 1998, p.340)

The above definitions agree that options, real or financial, are rights not obligations. The key difference among the definitions lies in the scope of real options, from assets in a narrow sense to actions in a broad sense.

4.2.4.2 Real Options Compared to Financial Options

Real options provide for the creation of value under uncertain conditions. Real options theory provides for a way of managerial thinking in three parts (Amram and Kulatilaka 1999): (1) options, real or financial, are contingent decisions – offering the opportunity to make decisions after events unfold, (2) option valuations are aligned with market valuations an on-par comparison of alternatives and transaction costs, and (3) option 'thinking' can be used to manage strategic investments proactively. Following initial identification and valuation, options offer the ability to redesign the investment and proactively manage the investment within the bounds of the reconfigured option.

Real options are similar in many ways to financial options: the initial decision is analogous to writing an option (i.e. buying or selling) and the subsequent decision to exercising (i.e. completing the transaction) or abandoning a worthless option. Real options emerge from the insight that many managerial decisions share common characteristics with decisions resolved by buying or selling options traded in financial markets (Janney and Dess 2004). Table 4.10 presents a comparison of the characteristics and definitions of financial versus real options.

However, there are two distinct differences between financial and real options. The information necessary to value a financial option and ultimately make the investment decision is readily available to all interest parties. Conversely, information surrounding real options is generally proprietary and unavailable to all but direct participants in the decision process (Copeland and Tufano 2004). Likewise, the value of the underlying asset of a financial option is generally known as are comparable values for like assets. The value surrounding the underlying assets on which real options are being considered are not clear and the value of like assets, when available for comparison, are often guessed.

Secondly, real option terms are often less clear that their financial counterpart. The ability to exercise a financial option is prescriptive, and the fulfillment of its transaction is executable in readily accessible markets. It is all too often unclear what the holder of a real option has the right to buy or over what period of time it may occur. Beyond duration uncertainty, real options are compound in that there may be sequential decision opportunities that uncover another option rather than the underlying asset, further hampering option clarity and timeframe.

Table 4.10: Comparison of Financial Option to Real Option (modeled after Janney and Dess 2004)

Item / Characteristic	Financial Option Application	Real Option Application
Writing an Option	A formal contract that sets the legal transaction terms	The initial decision that creates the opportunity to make a subsequent beneficial decision, there is no requirement for a formal contract
Exercising an Option	Formal activation of the legal terms of the contract as written	Subsequent beneficial decision made upon receipt of new information
Strike Price	Transaction price for the option – see also exercise price	The decision rule that informs / triggers subsequent decision(s)
Exercise Price	Price at which the option holder has the right to either purchase or sell the underlying asset	The cost of making subsequent decisions
Call Option	Option giving the holder the right to buy the underlying asset at a predetermined price	An option to enter a decision or a future decision; an option to defer
Put Option	Option giving the holder the right to sell the underlying asset at a predetermined price	An option to exit a decision or a future decision; an option to abandon
Liquidity / Tradability	Highly liquid asset – specific markets exist for the express purpose of option trade	Rarely liquid – difficult to trade, generally specific to company.
Timing	Pre-determined, precise, finite expiration date	Sometimes pre-determined, rarely precise, finite expiration date, can last indefinitely
Compounding	Two separate transactions, no compounding	A three or more part decision. Exercising one option creates additional future decisions
Portfolio	A collection of options	A collection of decisions
Underlying Asset	A financial instrument in the form of a stock, a bond, currency, a commodity, etc.	Tangible assets; may be intangible asset operating on or around a tangible asset

4.2.4.3 Real Option Taxonomy

As Dixit and Pindyck (1994), Trigeorgis (1996), and Amram and Kulatilaka (1999) have demonstrated, many decisions types are available with the use of real options theory. “The ability to value real options (e.g. to defer, expand, contract, abandon, switch use or otherwise alter a capital investment [the taxonomy of real option decisions]) has brought a revolution to modern corporate resource allocation” (Trigeorgis 1996, p.xi). Real options can be described in several dimensions (the source of ownership, the source of value, complexity, and availability (Ford et al. 2002)), and further defined by decision action.

More specifically, Boute et al. (2004) note that real options can be divided into analogous put and call options, whereby the value of the option remains the difference between the exercise price and the value of the underlying asset. A real option *call* is simply an option to defer the start of a project until additional information is available. Conversely, a real option *put* is the exact opposite – the ability to sell the asset underlying the project at the agreed upon exercise price (generally considered the project or asset’s salvage price).

Table 4.11: Correlation of Put / Call Options to Increase in Option Variables (modeled after Brealy and Myers 2000, Damadaran 2002)

Option Variable (an Increase in Value of)	Corresponding Value Changes	
	Call Option (Correlation + or –)	Put Option (Correlation + or –)
Asset value	Increases (pos. correlation)	Decreases (neg. correlation)
Exercise Price	Decreases (neg. correlation)	Increases (pos. correlation)
Interest rate	Increases (pos. correlation)	Increases (pos. correlation)
Time to Expiration	Increases (pos. correlation)	Increases (pos. correlation)
Volatility (change in uncertainty)	Increases (pos. correlation)	Decreases (neg. correlation)

Table 4.11 reflects the correlation of put and call options to the respective increase in the value of the option characteristic.

The taxonomy of real option puts and calls characterized by decision action are:

Option to Defer: An option to defer investment in a project or wait for a later time to start is similar to an American-style call option. It exists when there is uncertainty surrounding factors contributing to profitability and delaying a decision for additional information will not cause harm. An option to defer relies in flexibility in the form of upside potential of the decision, e.g. a decision to construct a facility based upon a real option to acquire land, the defer option is to wait to determine if there is a need and/or the ability to profit from actual construction).

Option to Abandon: An option to abandon is an option to exit the opportunity and is similar to an American-style put option. It exists when despite potential uncertainties and future risks, the decision to proceed with an investment in a project has been made, but conditions deteriorate such that it is advantageous to cancel the investment and recoup the salvage value. An option to abandon relies on flexibility in the form of downside protection of the decision, e.g. a decision to abandon capital-intensive investments like airline routes and speculative real estate development. In the case of development, the abandonment option is the sale of undeveloped land.

Option to Expand: An option to expand or grow an investment in a project is similar to both American- and European-style call options and is a compound option whose ultimate value depends on a pre-existing option. It typically exists when a project is phased into more than two stages, earlier options have been exercised and provides further opportunity for growth by the second or later investment. When considering the value of options to expand, it may be possible and/or advantageous to go ahead with the first project even if that project is expected to have a negative return. An option to expand relies in flexibility on the form of upside potential of the decision, e.g. a decision to research and develop a new product in the first phase, and then proceed with introduction of the product to the market in the second compound phase.

Option to Alter: An option to alter the state of an investment or project is an option to expand, contract, shut down or restart and is similar to both put and call options. A put option exists when there is the ability to respond to lesser demand and reduce planned operations or shutter them. A call option exists when additional capacity may be necessary and operations can be expanded, each either temporarily or permanently. An option to contract relies on flexibility in the form of upside potential and downside protection of the decision, e.g. a decision to increase or decrease production rates, particularly for natural resource-based commodities, based upon fluctuations in market demand and price.

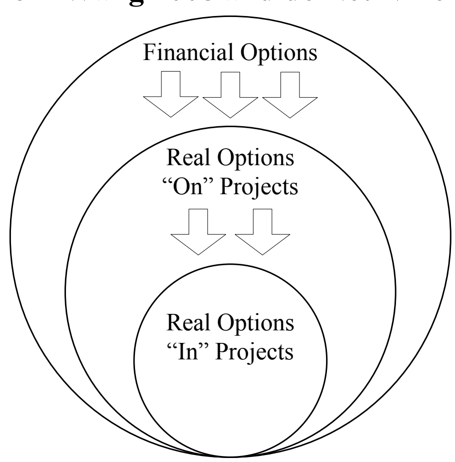
Option to Switch: An option to switch an investment in a project is an option to change either the inputs used in the investment or the outputs resulting from operating/completing the investment or project that were built into the initial design and is a combination of put options and call options. It exists when there is future uncertainty on either side of the investment, such that it becomes more advantageous to change the resource inputs or the resulting product output. An option to switch relies on flexibility in the form of upside potential and downside protection of the decision, e.g. a decision to switch between raw material source for the refining of oil, or conversely the decision to change the output product based from the refining operation, both based on market demand, on availability or price competition. In the case of real estate development, it may be the decision to switch between a condominium project and the development of a hotel.

Option to Time or Stage: An option to time or stage investment in a project is the option to segment the investment as a series of individual outlays to create both growth and abandonment opportunities and is a compound option similar to both American-style put and call options. It exists when there is future uncertainty in capital-intensive projects, particularly in start-up ventures. An option to stage or time relies on flexibility in the form of upside potential and downside protection of the decision, e.g. a decision to stage or time the investment in the development of pharmaceuticals, represented by an initial option to invest in the research and development for the initial drug and subsequent decisions to develop production facilities.

4.2.4.4 Real Options 'In' a System vs. 'On' a System

Unlike financial options, real options require physical structure, a project, or a system Figure 4.7 depicts the relationship between financial and real options. The purpose of the option remains the same: to allow investors the opportunity to purchase the right to delay costly or irreversible decisions. Like the ability to distinguish between real and financial options, there is also the ability to distinguish between two particular characterizations of real options: real options 'on' versus 'in' projects or engineering systems (Wang and de Neufville 2006). This differentiation appropriately separates real options in terms of identification, value determination, and complexity.

Figure 4.7: Option Theory Relationship
(adapted from Wang 2008 and de Neufville et al. 2005)



Most real options are options 'on' projects and simply refer to the aforementioned considerations. Real options 'on' projects and systems are similar to call options, as they represent the right, not the obligation, to invest in a project. Their analysis generally mirrors that of a financial call option. Real options 'on' projects consider the project / engineering system as a black box of sorts. That is, options 'on' projects are not concerned with the inner workings of the project, only the inputs and outputs when establishing an option value.

Real options 'on' a system afford the opportunity for four primary maneuvers when considering the decision to invest (Chambers 2007):

1. The right to acquire (to buy or begin) the project / system
2. The right to divest (to sell or abandon) the project / system
3. The right to expand the size of the project / system
4. The right to contract the size of the project / system

Each, while different, provides the option holder with the ability to defer important investment decisions until the information required becomes available, therefore helping to protect against uncertainty. Note that Chambers ignores timing / phasing in this context.

Establishing the value of the option would reflect the project specifics and correlate the value of the opportunity with the uncertainty. Most projects considering a real options analysis / valuation are unique, and the likelihood of finding a similar option after which to pattern and value is limited. Rather, an option value for a real option on a project / system can be constructed using the aforementioned valuation techniques.

Prior to the formalization of option theory, real or financial, opportunities were intuitively evaluated and ad-hoc options considered. The early benefit of these informal real options can be represented in maxims such as "a cunning rabbit has three caves," "never put all the eggs in one basket," and "a bird in the hand is worth two in the bush" (Wang 2008, p.98).

Conversely, real options 'in' a system are far more diverse, complex, and more difficult to identify and appraise. Their application must consider the inner workings of the project or engineering system. Real options 'in' a project or system originate from the system's design and entail an appropriate level of engineering knowledge. The decision to implement a real option ‘in’ a project or system will likely affect and be affected by other design decisions and maintain path-dependency (Chambers 2007). Real options that are ‘in’ systems or projects provide flexibility, and an option through the details of the design (de Neufville et al. 2004).

Real options ‘in’ projects are of particular interest to large complex engineering-based endeavors of the following characteristics (Roos 2004, Wang 2008):

- Long-lasting – to be designed with the demands of a distant future in mind,
- Exhibit economies of scale – particularly in large construction projects,
- Highly uncertain future requirements – recognizing that distant future forecasts are typically inaccurate.

These characteristics define the need for crafting project and/or system designs that can be easily adjusted over time to meet actual needs as they evolve. This positions real options ‘in’ projects as those that are the most interesting and challenging to designers. Real options ‘in’ projects and engineering systems are not to be confused with redundant design (Wang 2008). While both real options ‘in’ projects and redundancy suggest the presence of design elements that in a purely optimal design (one in which it is fundamentally assumed that uncertainty will not change things) should not have been included, redundancy refers to the duplication of design elements or components. Real option ‘in’ projects may not function as redundant elements for the purpose of increasing project or system reliability. Rather, they provide alternate paths for the project or system to traverse given an uncertain future, but like redundant elements, they may prove unnecessary should uncertainties not materialize. Table 4.12 presents a side-by-side comparison of the focus of real options ‘on’ systems to that of real options ‘in’ systems.

Table 4.12: Real Options 'On' versus 'In' Projects and Complex Systems (Wang 2008, Wang and de Neufville 2006, de Neufville et al. 2004)

Real Options "On" Systems	Real Options "In" Systems
Valuation of Investment Opportunity: To address whether investment in the project as a whole is worthwhile, e.g. valuation of oil fields, mines and other explorative ventures, and capital-intensive R&D programs, particularly pharmaceuticals	*Design of Flexibility within Project:* To address elements necessary to accommodate change in future direction or change in future events, e.g. design flexibility like additional fuel in satellites, spare tires in vehicles, increased capacity in structural systems, and the "bridge in a bridge / bridge on a bridge" concept for changing capacity traffic type and numbers of levels of a bridge in the future
Valuation Accuracy: Concerned with accuracy to assist with sound decision-making and in particular the value of the individual real option variations.	*'Go' or 'No-Go' Decision-Making:* Exact value less important and difficult to calculate; focus on what flexibility is designed into the project or physical system.
Easy to Define: Multiple types available: Options to defer, expand, contract, abandon, switch use, stage or time	*Difficult to Define:* Too many design variables within complex engineering systems and projects with the opportunity for each to generate an option; focus should remain on identification.
No Technical Knowledge Required: Evaluation does not require in-depth understanding of the inner workings of the 'black box' project; path dependency / independence not an overriding issue for concern.	*Superior Technical Knowledge Required:* Evaluation must carefully consider technological issues and the complexities of the design elements crafted as real options; path dependency / independence an issue for concern as the various design element constraints may for dependencies.

4.2.4.5 Real Option Strategy and Irreversibility

Real options can be used to decide between capital investments as well as for strategic decisions within the scope of individual projects (Bhargav 2004). The use of real options for capital investment decisions resolves which investment should be made, while the use of real options for strategic purposes addresses how flexibility should be used for decisions within a given project.

However, a real options approach is not always needed or appropriate. Some investment decisions become patently obvious as valuable and profitable (particularly of the 'black box' real option 'on' projects characterization), while others face the obvious recognition as sure losers, whereby real options considerations will not change the results. A real options analysis is of benefit in the following situations (Amram and Kulatilaka 1999):

- When contingent investment decisions / opportunities are available
  (no other approach can correctly establish a value in this situation).
- Under heightened uncertainty where it is of sufficient magnitude to wait
  for more information to avoid regrettable irreversible decisions.
- When the option value has been generated from possible growth opportunities
  and uncertain future cash flows rather than from current cash flows.
- When flexibility is required due to heightened uncertainty
  (only a real options approach can correctly establish a value in this situation).
- When the possibility exists for project updates and mid-course strategy changes
  (compound options).

Irreversibility is the strategic cornerstone of the Bernanke premise that uncertainty can increase the value of delaying decisions. Some investment decisions cannot be undone or substantially changed without incurring great or sometimes prohibitive costs. Irreversibility remains paramount in real option strategy and has been studied from multiple perspectives. Continuing Bernanke's economic analysis, Bertola (1998) concluded:

(1) Investment is not always positive if it is irreversible, (2) when an irreversible project is adopted, the ability to wait for uncertainty to be resolved is forsaken, and (3) [real] options are increasingly valuable even to risk-neutral investments as future uncertainty is amplified.

Irreversible investment decisions require careful initial analysis because, by definition once the decision to act has taken place, the investment cannot be undone without incurring significant cost. This necessitates delaying the investment until additional information is available, fulfilling the Bernanke premise and affording the flexibility offered by real options. "The value of an irreversible investment with its associated option is greater than recognized by traditional tools because the options truncate the losses" (Amram and Kulatilaka 1999, p.25). Typically, the use of real options will result in more irreversible investments, but in smaller stages after waiting for portions of uncertainty to be resolved.

Time matters the most when presented with an irreversible decision, but such decisions must be made with incomplete information. Accordingly, risk and time are diametrically opposed. They "are opposite sides of the same coin, for if there were no tomorrow there would be no risk. Time transforms risk, and the nature of risk is shaped by the time horizon: the future is the playing field" (Bernstein 1998, p.15). An early literary recognition of the time-risk conundrum and the need for real options was penned by Shakespeare (Bernstein 1998, p.15):

> "Hamlet complained that too much hesitation in the face of uncertain outcomes is bad because '*the native hue of resolution is sicklied [sic] o'er with the pale cast of thought... and enterprises of great pith and moment... lose the name of action.'* Yet once we act, we forfeit the option of waiting until new information comes along. As a result, not acting has value. The more uncertain that outcome, the greater may be the value of procrastination. Hamlet had it wrong: he who hesitates is halfway home."

4.2.4.6 Real Options, Arbitrage, and a Replicating Portfolio

In its purest sense, arbitrage is the simultaneous purchase and sale of an asset in different markets to profit from a difference in the price resulting from market inefficiencies (Webfinance 2011a). In practice, a profit made through an act of arbitrage involves buying an asset at a low price in one market and immediately selling it at a higher price in another market. The result is the realization of risk-free profits made with no net investment, because the cash received from the sale is sufficient to finance the purchase (Moles 2006). Arbitrage opportunities rarely exist. According to the *Law of One Price* (Lamont and Thaler 2003), in efficient markets, all identical goods must have only one price.

The expectation for 'no arbitrage' is a condition precedent to financial options and by extension in part to real options. It predicts that assets with the same risk profile will trade at the same price irrespective of market specifics. By theory, a complex investment opportunity (as may be found in a real option) can be replicated with an equivalent portfolio of simple financial instruments, each of which can easily be priced. This 'replicating' portfolio and the option in the absence of an arbitrage situation will result in the same payoff and therefore must have the same price.

The no arbitrage condition allows for the pricing of real options with the use of a replicating portfolio of substitute financial instruments that provide the same payoff as the underlying asset of the real option, provided they are directly observable in tradable financial markets (Dixit and Pindyck 1994).

However, the theory of real options faces conceptual criticism and practical shortcomings. The elegance of financial options and the no-arbitrage enforced price equality is ubiquitous and easily demonstrated. This relevance of the no-arbitrage capacity is difficult to see in many cases with real options. Absent a defined marketplace for exchange and ease in pricing, coupled with the subjective nature of selection of discount rates and the equally subjective and potentially arbitrary assessment of the value of risk, implies that real option analysis cannot be objectively based on market prices. It further suggests that those pricing real options and those acquiring them can maneuver the analysis / valuation process to achieve predetermined results.

Differing results and conclusions can be reached and be purported accurate by each analysis, because there is no market for direct comparison, nor any potential comparative accuracy. Despite these challenges and resulting doubts, real option theory has gained acceptance and is widely practiced. As noted by German philosopher Hegel, "Whatever is reasonable is true, and whatever is true is reasonable" (Wang 2008, p.129). This explains the real option analysis versus no-arbitrage / definable market paradox and why real options are powerful: their reasonableness in application to unique project opportunities.

4.2.4.7 What Must Exist to Value Real Options

Real options theory cannot be applied to every scenario surrounding an asset-based project decision. Certain criteria must exist for a real option to become viable. According to Mun (2006), for a real option analysis to become viable, the following five conditions and/or requirements must be present or must be satisfied in the future:

Financial Model: A financial model depicting discounted cash flows from the project or opportunity must exist. Real options build upon such an analysis from prior strategic decision-making and planning. Absent such a model, the decisions have already been made, and the need for real options against future uncertainty is moot.

Uncertainties: Without future uncertainty, any option is worthless and an exercise in futility. If uncertainty has been removed from the process, the aforementioned discounted cash flow model is sufficient. By definition, when uncertainty / volatility is zero, the value of the real option is zero, and the resulting option valuation method returns the same value as the discounted cash flow model (a standard NPV calculation).

Affected Decisions: Uncertainty must affect future decisions when actively managing a real asset-based project or opportunity, and the uncertainties must directly affect the results of the financial analysis. These uncertainties will directly correspond to future risks for which options can be crafted to hedge / overcome their impact.

Flexibility: Decision-makers must have the ability to change course via active management by strategic flexibility or options to execute when uncertainty-driven risks materialize.

Ability to Decide: Decision-makers must have the ability, credibility, and opportunity to exercise the option and implement the resulting changes when it becomes optimal to do so. Absent the intelligence to appropriately execute at the right time and under the right conditions, valid real options become worthless.

4.2.4.8 Valuation Methods for Real Options

Drawing from financial options theory and its foundation in capital budgeting techniques, there are multiple ways to price real options. The application of these methods coalesce around three generic valuation methods: the market approach, the income approach, and the cost approach (Mun 2006):

Market Approach: Valuation through the use of comparable assets in the marketplace and their corresponding price, a replicating portfolio approach. It assumes a rational market and a no arbitrage situation. The market approach also requires adjustment or normalization at the market level, at the industry level, and at the organization or firm level.

Income Approach: Valuation through the analysis of future potential profit of free cash flows generated from the underlying asset, a basic net present value (NPV) calculation to discount at a predetermined rate (either a firm-specific hurdle rate, the weighted average cost of capital, a risk-adjusted rate, the market risk-free rate, an expected rate of return, etc.) the opportunity-specific cash flow stream.

Cost Approach: Valuation by looking at the cost incurred to replace or reproduce by other means the underlying asset's future profitability. The cost approach requires the inclusion of the intangible side of the underlying asset's development, the strategic elements used as if the asset / opportunity were developed from the ground up.

From within these approaches, three primary valuation methodologies rise to the level of importance in pricing real options: Black-Scholes Equation (Eqs. 4.11 and 4.12), simulation and the binomial lattice method from decision theory (Wang 2004). In theory, these techniques should produce the same result for a real option analysis as they do for financial option valuations. However, given differing real option circumstances (real options being unique to the opportunity and organization), one valuation method may become more applicable to the specific option being analyzed than the other.

Wang (2004) concludes that depending upon the specifics, selection of a single valuation approach should consider the following:

- The Black Scholes method should be employed with great care given the assumptions associated with its application: the availability of a market for trading, an understanding of market prices, efficient markets with the no-arbitrage condition, the ability to assemble a replicating portfolio, and a uniform risk-free interest rate.
- Simulation though useful has limitations for which variance reduction techniques become required to obtain the necessary accuracy and value convergence. Using the wrong modeling techniques or parameters can cause erroneous results and lead to expensive computational processes for which a single value is produced absent evaluation / consideration of the intrinsic relationship between the variables being simulated.
- Binomial methods remain versatile, but path dependency must be examined to determine if the tree configuration (path dependent) or the lattice variation (path independent) is appropriate. While financial options remain binomial, real options can be trinomial or more, negating the advantages of the recombining lattice and its reduced node quantity.

4.2.4.9 Challenges with Real Option Analysis

The application of real options presents many uncertainties, challenges, and problems. Real options are most often a custom application representing the decision and organization at hand, lacking direct parity with the antecedent financial option. In particular, the underlying asset as well as the real option itself may lack the ability to be traded. Beyond this, problems with real options analysis also arise from the use of valuation models that demand more simplicity and clarity in their application than the real options world presents (Copeland and Tufano 2004). For example, the Black-Scholes valuation model, originally developed to price a European-style option without any dividend payments, was never intended for use with more complex financial derivatives and options, and its application to real option valuation may present problematic results and lead to inappropriate conclusions.

A powerful tool when faced with an uncertain future, real options cannot solve all of the problems associated decision-making under uncertain conditions (Janney and Dess 2004). Failing to understand their limitations or mischaracterizing their value can lead to an unsupported confidence in the resulting decisions. Some of the pitfalls associated with real option analysis surround net present value (NPV) calculations, agency theory, managerial adventurism, and portfolio theory (Janney and Dess 2004, Table 1). Similarly, real options 'in' versus 'on' projects present differing challenges.

Net Present Value Difficulties: A key element in the development of a real option model is statistical variance. Variance is a formal way of measuring uncertainty and deciding between options: the best case versus the worst case. Variance can only be measured for past occurrence, not for future decisions. Variance for future decisions must be estimated and such estimates being subjective lead to widely changing results from the slightest change in the estimated variance. A low variance (equating to low expected fluctuation in outcome and equally low yields), typically results in a decision not to proceed. Conversely, high variability within a model produces positive decisions.

The same can be said for NPV calculations and the selected discount rate. Slight adjustments in the rates generate large swings in the net present value of the real option opportunities, either positively or negatively, and may impact decision-making inappropriately. Decisions surrounding sensitive outcomes are greatly impacted by subjective inputs.

Significant differences between the composition of financial and real options also impact NPV calculations. Financial instruments are liquid; they can be traded an almost any moment in their life. Real options are mostly non-transferable (illiquid) and suffer from company uniqueness (Janney and Des 2004). Variance estimates become problematic absent an opportunity to trade options. Likewise, financial options have expiration dates that render an option valueless if not exercised. Real options may have indefinite lives and a positive lifetime value even if not exercised (the value of the underlying asset, such as a parcel of undeveloped land, remains the property of the option holder irrespective of option status).

Of unique concern and most problematic to NPV calculations is determining the value of an indeterminate option component or type, e.g. the NPV of an investment in learning or something that has never happened or been attempted. Modeling variance estimates, including NPV calculations, also become difficult when an option holder has no prior experience in decision-making with similar opportunities and thereby lacks a history of variance estimates or discounts rates that have proved their applicability.

Additionally and more importantly, traditional NPV calculations lack the ability to address flexibility. A pure NPV calculation assumes Bernanke's condition of irreversibility; the decision is a now or never opportunity, and if not taken, will be lost. NPV does not factor that decisions surrounding real options may be implemented flexibly through deferral, abandonment, expansion. They may be phased, grown, switched, or compounded (Wang 2008).

Agency Theory: The agency leadership concept at work in most businesses involves the trust placed in individuals to lead / manage an entity primarily owned by others, e.g. publically traded corporations. An agency problem arises (conflict exists) when leaders or agents' operate in their own self-interest, rather than in the interest of the owners. Agents may have personal ambitions that compete with the ownership's desire to maximize its investment. As owners have entrusted agents with decision-making and administration of the organization, a potential conflict of interest exists between the two groups. Agency theory concludes that when presented with this situation, agents will follow the path of their own self-interest and benefit.

When properly trained in valuation techniques, managers have the ability to 'back-solve' a valuation exercise to arrive at a predetermined result or range of results. If the hurdle rate is known or the aforementioned capital budgeting technique expectations are established, it is easy to create a valuation model to deliver those results. That is, the valuation system upon which real option decisions are founded can be 'gamed.' Managers have an inherent interest in the subjective choice of variance values (discount rates, etc.) that increase the likelihood of increased performance and approval, ultimately the exercise of the real option.

Conversely, managers with responsibility for exercising real options face an opposite agency problem. Decisions to exercise real options require a greater commitment and ongoing involvement than those to abandon, as well as more effort than that required to develop the option. Agency theory suggests that decision-makers will tend towards the least involved path forward.

Overconfidence and the Illusion of Control: In a form of managerial adventurism, managers face a myriad of challenges that can impact the decision to exercise a real option. Those that have had successful opportunities with real option decisions in the past may tend to believe that they possess superior skills or knowledge. This leads to ill-informed decisions, ones based on the illusion of instinct and the belief that risks can be overcome by one's own involvement or expertise, versus analysis-based decisions.

Whether large or small, the real option decision is designed to minimize risk and maximize future gains. Adventuresome managers tend towards activity and the proclivity to proceed with the real option decision at hand in a careless manner, with the expectation that any risks will be minor and easy to solve. "Thus, managers may approach each real option decision with less care and diligence than if they had made a full commitment to the larger investment" (Janney and Dess 2004, p.68).

Portfolio Pitfalls: By definition, real options minimize downside risk, the risk and losses associated with negative consequences and decisions. Managers who are responsible for multiple real options can be faced with numerous individual decisions that have downsides. That is, they have the opportunity to make multiple decisions that result in small losses. When aggregated, these losses may exceed the amount of any single loss that the portfolio manager was willing to accept. Therefore, a portfolio of real options may lead to greater risk, not less.

Much like multiple real options, multiple decision-makers can negatively impact the valuation and exercise of real options. If decision power rests with multiple parties having disparate or no cost, it can result in decisions to move forward that are not in the organization's best interest. Ultimately, this could waste valuable resources and lead to a *tragedy of the commons* issue, a dilemma arising when multiple individuals acting in their own self-interest will deplete a shared limited resource, even when it is clear that it is not in anyone's long-term interest for this to happen (Harden 1968). Those maintaining decision control, without any consequence to their action, only upside benefits, are more likely to exercise than to abandon a real option. In this situation, emphasis remains on outcomes rather than on process.

4.2.4.10 Valuation Elements for Real Options

Much like financial options, there are five elements required to value a real option: (1) the value or price of the underlying asset, (2) the exercise or strike price, (3) the time to expiration, (4) volatility, and (5) the risk-free rate (Frayer and Uldere 2001), as summarized in Table 4.13.

Value or Price (S): The value of the underlying asset (stock, bond, etc.) on which an option is purchased. It is simply the market's estimate of the present value of all future cash flows, (dividends, capital gains, etc.) associated with the asset. Its equivalent in a real option valuation is the present value of cash flows expected from the opportunity on which the option is sought.

Exercise or Strike Price (K): The predetermined price at which an option can be exercised. Its equivalent in the real options arena is the present value of all the fixed costs expected over the lifetime of the opportunity, i.e. simply the investment cost of the underlying real asset.

Time (T): The period during which the option can be exercised. Its equivalent with respect to a real option is the period for which the opportunity remains valid until a decision must be made or the opportunity will disappear.

Risk-free Rate (r_f): Typically the yield of a riskless security with the same maturity as the duration of the option. It is the same with regard to financial or real option valuations.

Table 4.13: Variable Comparison between Financial and Real Options (modeled after Trigeorgis 1996, Ramírez 2002, Menassa 2007)

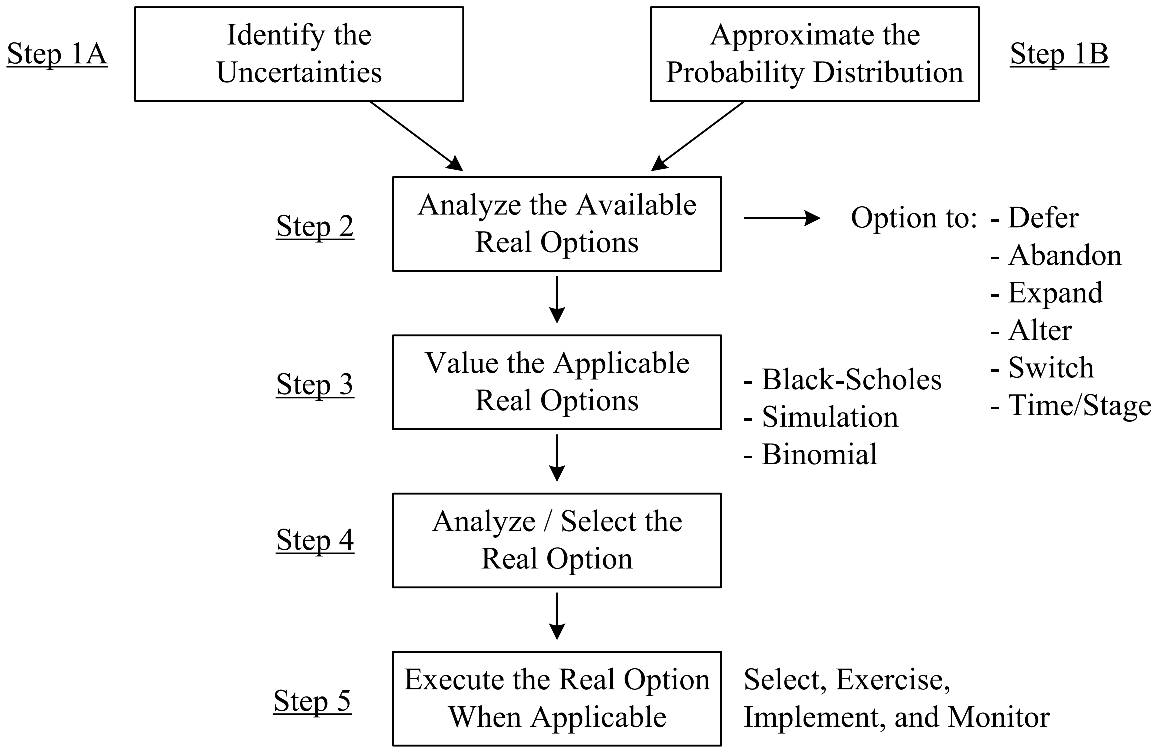
Element	**Financial Options**		**Real Options**	
	Variable	**Equivalency**	**Variable**	**Equivalency**
Value	S	Current Stock Price	S	Present Value of Asset
Strike Price	K	Exercise Price	K	Investment Cost
Time	T	Time to Expiration	T	Time Until the Opportunity Disappears
Rate	r_f	Risk-Free Interest Rate	r_f	Value of Money Over Time
Uncertainty	σ_s	Stock Value Uncertainty	σ_r	Project Value Uncertainty

Uncertainty or Volatility (σ): The measure of the unpredictability of future price movements, i.e. the uncertainty of future asset value. For financial options, it is the standard deviation of the growth rate of the value of future cash inflows associated with the asset (the stock). The real option equivalent is similar in that is relates to the uncertainty and ultimate value of the cash flows associated with the asset.

4.2.4.11 Real Option Valuation Process

Wang (2008) identifies a multi-step process to arrive at the value of a real option on a project. Real options 'on' project value the project as a black-box and do not venture into the realm of the interworking of the project; the value is based on the action being taken (or not taken) with respect to the underlying asset. Figure 4.8 depicts the modified steps to the valuation process as follows:

Figure 4.8: Real Option Valuation Process
(adapted from Wang 2008 and Koller et al. 2005)



Step 1: Identify the most important drivers of uncertainty in the project or opportunity. Uncertainty is twofold (internal and external to the organization) and includes market risk (pricing, supply and demand constraints, market strength / economic conditions, etc.) and technical risk (schedule constraints, budget adherence, project / product performance to expectations, etc.). Approximate the probability distributions for each uncertainty identified, as well as for any other project / organization-specific risks identified on a case-by-case basis.

Step 2: Indentify the real option types available and applicable to the opportunity at hand: options to: defer, abandon, expand, alter, switch, and time or stage.

Step 3: Value the real option by choosing among and applying the aforementioned methods to arrive at the value of the options. This is the value of the overall real option, not the flexibility component nested within the real option 'on' a project or opportunity.

Step 4: Analyze the real valuation results, pairing for comparison the option value and the cost to obtain the option.

Step 5: Select the most worthwhile / appropriate option, exercise, implement, and monitor it.

Of concern to the real option valuation and implementation process is the perception of precision. As discussed above, multiple elements within the valuation process are subjective. They lead to a false perception of accuracy and/or precision in the resulting values. Sensitivity analysis is often needed and can be performed useing of multiple valuation methods for the same option. While it would be expected that each method would generate the same result (as is the case with financial options), the subjective nature of the valuation process and the uniqueness of the opportunity provide the possibility for disparity in results between real option valuation methods.

4.2.4.12 Valuing the Flexibility within Real Options

Nested within the value of a real option is the flexibility associated the ability to postpone the decision and/or take multiple paths in its execution. This is the Bernanke (1983) supposition that postponing a decision, while maintaining the ability to commit at a later time, can prove desirable by allowing choice only after important information is revealed, and such ability has inherent value. Koller et al. (2005) present a method for determining the specific value of flexibility associated with a real option, depicted in Table 4.14 and described below.

Estimate NPV without Flexibility: Conduct a valuation of the project / opportunity without the advantage of flexibility in decision-making by using a traditional discounted cash flow method from the aforementioned capital budgeting valuation models of corporate finance – with net present value being the foremost of them.

Model Uncertainty in an Event Tree: Expand the discounted cash flow valuation into an event tree (a decision lattice), specifically mapping how the project / opportunity changes over time. This step does not yet incorporate flexibility into the valuation model, so it should result in the same value as the NPV calculation.

Model Flexibility in Decision Tree: Convert the event tree (decision lattice) into a decision tree with the inclusion of decision flexibility as available under the real option scenario or scenarios available. Multiple forms of flexibility may be available at a single decision node (akin to compound options) within the decision tree like the opportunities to abandon or expand, so it is paramount to prioritize the options.

Estimate the Contingent NPV – The Value of Flexibility: Recognize how the inclusion of flexibility in decision-making changes the project risk characteristics by discounting the cash flows at each decision node, using the risk-free rate for investment cash while using a risk-adjusted rate (such as the weighted average cost of capital for the organization, or other appropriate market related risk-recognizing rate) to discount the resulting project cash flows.

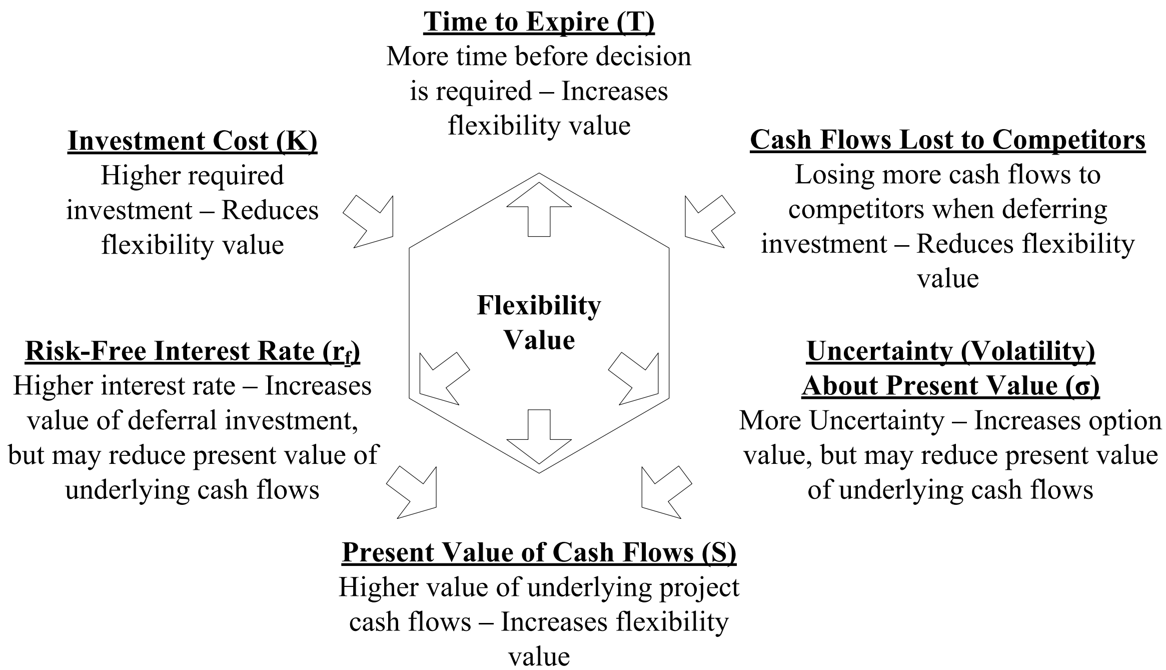
Table 4.14: Process for Valuing Flexibility (Koller et al. 2005, p. 560)

	Estimate NPV without Flexibility	**Model Uncertainty in Event Tree***	**Model Flexibility in Decision Tree**	**Estimate Contingent NPV**
Objectives	Compute the base case present value without flexibility	Understand how present value develops with respect to changing uncertainty	Analyze event tree and incorporate managerial flexibility to respond to new information	Value total project using decision tree analysis or a real option valuation approach
Comments	Standard NPV approach used for valuation of the underlying asset	No flexibility modeled, valuation using event tree should equal standard NPV	Flexibility is incorporated into the event tree* (decision lattice), transforming it into a decision tree	Under high uncertainty and managerial flexibility, contingent NPV will be significantly higher than the standard NPV

* The event tree described by Koller et al. is the same as the decision lattice described herein.

In conjunction with this, Koller *et al.* also identify the specific drivers of the value of flexibility (Table 4.13) as associated with the valuation elements for real options (Figure 4.9).

Figure 4.9: Drivers of Flexibility
(Koller et al. 2005, p. 549)



4.2.4.13 Misunderstandings of the Application of Real Options

The application of real options theory to projects and other organizational opportunities is not without its detractors and critics. Several criticisms surrounding the use of real options analysis are found, typically when being applied by the novice or ill-informed analyst. Mun cites three misunderstandings about real option application. Real options analysis can be incorrectly classified as (Mun 2006):

- *An Impractical Academic Exercise:* Real options analysis is merely an academic exercise and has no direct practical application in business.
- *An Inflationary /Incorrect Valuation Method:* Real options analysis is a means to incorrectly increase the value of an opportunity to justify implementation.
- *Predisposed to High Risk Projects:* Real options analysis results in the selection of opportunities with the highest risk profile. The higher the volatility, the higher the option value.

4.3 Analogy to Construction Network Schedules

4.3.1 Correlation of Uncertainty and Flexibility to Float

Disparate entities, herein general contractors and their subcontractors, participate in a complex decision-making process subject to constraints and uncertainties that determine whether a project will be completed on time. These uncertainties precipitate the need for accurate and timely information upon which to base decisions, particularly for schedule matters. Such decision-making processes afford selection among multiple options and project opportunities, for which the ultimate expression is the consumption of float.

Float is a measure of flexibility that reduces risk and increases opportunity (Thompson and Lucko 2011). Its various types quantify the ability of an entire schedule or individual activity to accommodate uncertainty and absorb delays. Real options become *apropos* to schedule systems with float being the measure of flexibility within the systems and uncertainty being the driver of and necessitating flexibility. The Bernanke concept of waiting for additional information is appropriate to the multiplicity of activities and the uncertainties surrounding their completion.

4.3.2 Flexible Decision Structures within Construction Projects

Construction project planning efforts include the identification, assessment, and selection of alternative strategies (Ford et al. 2002). This includes the management of uncertainty throughout the planning and construction periods. Construction projects and activities evolve over time, for which the conditions cannot be fully determined, accurately described or accommodated, so that uncertainties of varying size materialize, often too vague and disguised for effective mitigation.

Construction project planners are faced with the dilemma of needing to address uncertainties before proceeding, but lack the strategies and tools to value the increasingly dynamic uncertainties of projects during pre-planning and estimating endeavors. The knowledge about future conditions that is needed to make efficient and effective decisions is either unavailable or inadequate during the planning stage (Ford et al. 2002): "[T]he effective management of these high impact dynamic project uncertainties can increase project value. The ubiquity and potency of dynamic uncertainties [one where the cost of selecting suboptimal alternatives during pre-construction activities is high] require that they be managed effectively if all the value in a given project is to be developed and captured" (Ford et al. 2002, p.344). Construction project managers are aware of the potential benefits of uncertainty in projects, e.g. price uncertainty against fixed-price contracts. They must be able to demonstrate the benefits of applying a real options approach to the planning of projects and identify the implementation challenges. The use of a real options approach to the planning and control of construction projects can have the following impacts (Ford et al. 2002):

- *Increased Awareness:* An increased description, measurement, and management of the uncertainties found within construction projects, and greater perception of uncertainty generated opportunities and corresponding risk.
- *Increased Involvement:* More purposeful planning and increased managerial flexibility resulting in improved control over project constraints to capture value.
- *Increased Competitiveness:* Added value through managerial agility surrounding uncertainty and the ability to capture latent value.

Ford et al. conclude that the strategic application of real options to construction projects to manage uncertainty can maximize project value. Uncertainty carries negative connotations, focusing on project losses and schedule delays. Project managers tend to limit their efforts to the mitigation of uncertainty and its undesirable impacts while ignoring hidden and unexploited project value (Bhargav 2004).

Unlike options on traded financial instruments, real options involve the application of the pricing theory of capital investing decisions (Boute et al. 2004, Trigeorgis 1993) that shares similarities with the uncertainty-driven decision-making needs of construction projects. From a strategic perspective, option theory can be directly extended to construction projects. Ford et al. (2002, p.346) posit that "[a] real options approach in construction projects improves strategic thinking by helping planners and organizers recognize, design, and use flexible alternatives to manage uncertainties." They further conclude that construction projects as a whole can be described as real options, as can individual activities and tasks therein, a loose extension of the difference between real options 'on' versus 'in' systems.

4.3.3 Real Option Extension to Construction Projects

The framework of real option investments, characterized by sequential, irreversible investments made under conditions of uncertainty (Dixit and Pindyck 1994), suggests that purchasing a real option on a strategically important opportunity allows the postponement of a commitment (or of a decision) until a substantial portion of the uncertainty or risk surrounding the opportunity has been resolved (Adner and Levinthal 2004). Real option use fulfills the Bernanke (1983) precept that postponing a decision, while maintaining the ability to commit later after important information is revealed has inherent value. Boute et al. (2004, p.9) validate the value of real option strategy in construction projects, "[i]t is inherently clear that the longer the contractor waits, the more additional information he obtains and thus the more valuable the option will be."

As options afford the financial market and real property decision-makers the ability to manage risk, so too does float afford network schedule system participants the ability to address schedule risk. The exercise of an option is the vehicle by which risk is priced and mitigated within financial markets and real asset projects / opportunities. Therefore, the decision to expend float is the analogous exercised real option within schedule systems.

The conceptual difference between financial and real options and construction projects is that activities in a construction project and the resulting decisions occur within a closed dependency structure per the schedule network and project at hand, whereas financial and real assets forming the underlying asset for which an option is purchased can be tradable within neutral organized markets.

4.3.4 Real Options and Network Schedule Systems

"The flexible decision structure considered in option theory is also valid in scheduling" (Boute et al. 2004, p.2). Uncertainty surrounds the timely completion of construction projects: Similarly, the required resources and their availability, as well as the expected duration, may be undetermined when proceeding with a project. These uncertainties may lead project managers to wait for more or better information in development of an initial schedule, when modifying a previously determined course of action, or when completing work. Using traditional techniques, such as net present value and/or decision tree analysis, when planning and scheduling work in or under uncertain conditions may lead to false and/or misleading results. "Instead, a real options analysis should be used" (Boute et al. 2004, p.2). A real options analysis overcomes the flaws of discounted cash flow methods, e.g. subjectivity with respect to the discount rate used, the predisposition to use average cash flows, a lack of consistency as to which costs on include or exclude, etc.. "[I]t explicitly recognizes the value of flexibility and the additional value associated with options in the context of uncertainty, especially when system operators [project management] can manage these uncertainties" (Bhargav 2004, p.14).

4.3.5 Quantifying Uncertainty for Real Options Applications in the Construction Industry

Construction projects are notorious for time and cost overruns (Creedy et al. 2010, Shane et al. 2009, Kim 2007, Georgy et al. 2000), in part due to a "deficiency in managing the scope, time, quality, cost, [and] productivity" (Jergeas and Ruwanpura 2010, p.40). Depending on project type, schedule variation (a loose indication of uncertainty and/or volatility) ranges from moderately early finishes to far exceeding planned durations. An international study of over 200 building projects calculated that approximately one-third finished on or ahead of planned duration (Acharya et al. 2006), with 20% exceeding their planned duration by more than 50% (Table 4.15 depicts the full schedule variability findings of Acharya et al.). Further construction project volatility is portrayed by Bhargava et al. (2010). In a 1,800-plus highway construction project study, almost 90% of projects exceeded the planned construction duration.

Like schedule overruns, construction costs tend to exceed originally estimated and contracted amounts. A Flyvbjerg et al. (2002) global infrastructure project study found that over 50% of large projects overran their initial budget, and on average, a cross section of international transportation projects exceeded their cost estimates by nearly 30%. Shane et al. (2009) describe construction project management as the coordination of a multitude of human, organizational, technical, and natural resources, the complexities of which can be overshadowed by economic, political, and societal challenges. Challenges as such often influence project cost escalation, generating volatility (the surrounding amount of uncertainty or risk). Such volatility is typically measured by the standard deviation or variance (in this instance 38.7% per Table 4.16).

Table 4.15: Building Project Duration Delay (Acharya et al. 2006, Table 4)

Schedule Delay	Percent of Projects
Early Finish	5.5%
No Delay	28.9%
Under 10%	7.0%
11 - 25%	14.8%
25-50%	23.4%
Over 50%	20.3%

Table 4.16: Inaccuracy in Project Cost Estimates (Flyvbjerg et al. 2002 Table 1)

Project Type	Average Cost Escalation	Standard Deviation (σ)
Rail	44.7%	38.4%
Bridge	33.8%	62.4%
Road	20.4%	29.9%
All	27.6%	38.7%

The volatility of construction material costs adds to overall industry uncertainty. Massachusetts Institute of Technology studied the real price changes of four basic construction materials: concrete, asphalt, steel, and lumber. Nominal price indices between 1977 to 2011 were obtained from the Bureau of Labor Statistics (BLS) Producer Price Index and adjusted by the overall Consumer Price Index to arrive at real costs (Lindsey et al. 2011). The four materials had different statistical attributed as depicted in Table 4.17.

Table 4.17: Mean and Standard Deviations of Annual Percentage of Real Price Changes (Volatility) (Lindsey et al. 2011, Table 3)

	Concrete	Asphalt	Lumber	Steel
Average	-0.17%	1.25%	-1.20%	-0.16%
Standard Deviation (σ)	2.19%	6.3%	8.9%	8.9%

From a financial perspective, project uncertainty can be measured by the variation in cash flow streams. Cobb and Charnes (2003) evaluated the expected rates of return from project cash flows. With the aid of simulation techniques, and the recognition that "estimating the volatility parameter for a real options model is difficult because neither observed historical returns for the underlying real asset nor current market prices are available…and there are typically no historical returns for assets that are perfectly correlated with the project cash flows," Cobb and Chanes (2004, p.6), modeled five year cash flow performance to determine rate of return (ROR) distribution for which a frequency and probability distribution could be developed. Statistical analysis generated a ROR standard deviation of 35.35%; "which will be used as the volatility estimate" in a real options analysis (Cobb and Chanes 2003, p.15).

Uncertainty surrounding construction project completion remains relatively high. Schedule variation in the negative sense (extended durations) are commonplace and impact between 65% and 90% of projects. Upward cost pressures compound uncertainty and are present in approximately 28% of transportation projects with a volatility measure (standard deviation) of 38.7%. Material costs have annual price volatility as measured across four commodity materials (Table 4.17) ranging from a standard deviation of slightly above 2% to approximately 9%.

Projects in which trading float becomes desirable experience greater levels of uncertainty, activities are need to address schedule delays by some means. Determining the appropriate uncertainty and/or volatility measure to implement in construction project real options analysis has not been addressed, nor is any measure like the ubiquitous financial beta (β), the measure of the volatility, or systematic risk of a security (an individual company) in comparison to the market as a whole, from the Capital Asset Pricing Model of portfolio theory (Black, Jensen, and Scholes 1972) available.

Establishing the appropriate uncertainty measure (σ) for construction projects currently is more of an inference than a deterministic process. By assimilating schedule uncertainty, cost volatility and material price fluctuations, and comparing them to other industries and modeled cash flows, uncertainty can be assumed. Variation measures relative to construction projects range from 2% to nearly 40%, and there is a high probability for schedule delays. Therefore, an assumed uncertainty (σ) between 25% and 50% appears reasonable.

When compared to the oil and gas industry, where real options project analyses are commonplace, which employs oil and gas price fluctuation as the measure of uncertainty in real option valuation, σ ranges between 30% and nearly 65%. Over 12 years for which multiple data sources were available, it was demonstrated that U.S. oil prices experience high levels of volatility, reaching as high as 64.5% around the mean barrel price, but generally settle around the long-term historic price and annual volatility of 29.5% (Piesse and Vad de Putte 2004). This corresponds to the uncertainty modeled for project cash flows and that assumed for construction projects.

4.3.6 Real Option Methodology for the Valuation of Float

The real options component of decision analysis, which effectively takes into account the value of flexibility by structuring the problem such that the uncertainties and contingent decisions are represented by a decision tree (Bhargav 2004), presents distinctive structural parallels to network schedule systems. Whereas a decision tree is a sequence of decision and chance nodes, a network schedule system is a series or collection of sequences, terminating at a single activity (or node). A decision node indicates a point where the decision-maker faces a decision, much like individual activities and their interrelationship with predecessor, successor, and/or parallel activities that necessitates ongoing decisions to reach fruition.

Similarly, but not directly parallel, the branches emanating from a decision node represent the options available to the decision maker that must be mutually independent, while the successor activities within a network schedule system maintain a dependency relationship. By definition, one must achieve completion before the next may begin. Capturing the value of the decision to expend float, the option at hand, fits well with the binomial nature of the decision tree structure.

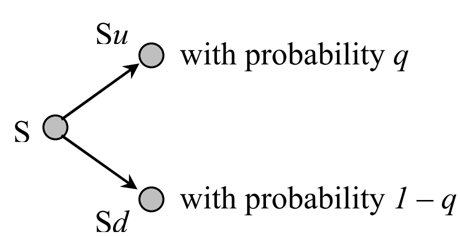
Decision-tree analysis fosters multi period development and evaluation of a growing number of options. The extension of such analytical methods to network schedule systems is similar but remains a single period function at each schedule delay occurrence. The failure to meet the scheduled completion of an activity calls for a binomial decision: (1) expend float where available to mitigate the schedule impact, or (2) overcome by other means, e.g. accelerate the delayed activities (also considered for clarity herein as including an overall schedule delay found to be acceptable). This does not preclude the extension of decision tree / real option binomial pricing methods from being used in the valuation of float at individual opportunities within network schedule systems. It merely fails to fully utilize the advantages of the multi-period calculation power of decision trees and the reduction of complexity attributed to the ensuing decision lattice structure.

4.3.7 Binomial Calculation Methods

Real option calculations require several inputs (Table 4.13), with those applicable to the binomial method being: (1) present value of underlying asset, (2) investment cost, (3) time, (4) time value of money – the risk-free rate, and (5) uncertainty or volatility. Investment cost or strike price (*K*) (the cost of purchasing the option) is necessarily considered in all applications of model). The extension of these determining elements to the valuation of float within construction project network schedule systems follows the binomial methodology put forward by Cox et al. (1979):

Their model consists of a binomial lattice that depicts two possible changes in value for an underling asset, i.e. stock over sequential time periods. A move up by a factor *u* or a move down by a factor *d* (Figure 4.10), incorporates a risk adjustment that is founded upon the probability of an upward move being represented by *q* and the downward move probability being *1 – q*, corresponding to the probabilities that a risk-neutral investor would assign to the two outcomes.

Figure 4.10: Binomial Decision Diagram



There are multiple ways to estimate binomial movements and the associated risk neutral probabilities, all of which incorporate the uncertainty or volatility associated with the project. Cox et al. define the up (*u*) and down (*d*) movements at each decision node by Eq. 4.20 and Eq. 4.21 respectively (Hahn and Brando 2010):

$$u = e^{\sigma\sqrt{\Delta t}} \qquad \text{[Eq. 4.20]}$$

$$d = e^{-\sigma\sqrt{\Delta t}} \text{ or } \frac{1}{u} \qquad \text{[Eq. 4.21]}$$

where

σ = the volatility of asset returns
Δt = the portion of time period T being evaluated (applicable part of 1 year)

With u and d quantified, the probability q for an upward move is then (Cox et al. 1979):

$$q = \frac{(1 + r\Delta t - d)}{(u - d)} \qquad \text{[Eq. 4.22]}$$

The corresponding probability of a downward move is simply $1 - q$. This is predicated upon the assumption of that the values u, d, and q over time follow Geometric Brownian Motion. Cox et al. further demonstrate that this approach remains valid with either type of binomial decision vehicle: decision trees or the recombining decision lattice.

Consider the project investment described in Table 4.18, a two-period decision tree (two six-month periods) with an initial $100 investment opportunity (absent any acquisition costs) and an assigned volatility at 0.20:

Table 4.18: Binomial Option Example Calculation Inputs

Option Input	Variable	Value
Present Value of Investment	S	\$100
Investment Cost	K	N/A
Time Period	Δt	6 months (0.5-year)
Value of Money Over Time	r_f	5% / year (2.5% / period)
Volatility / Uncertainty	σ	0.20

The resulting upward and downward movements as determined by Eq. 4.20, Eq. 4.21, and Eq. 4.22 respectively, for which the resulting values are depicted in Figure 4.11 and discounted at the annual risk free-rate of 5%:

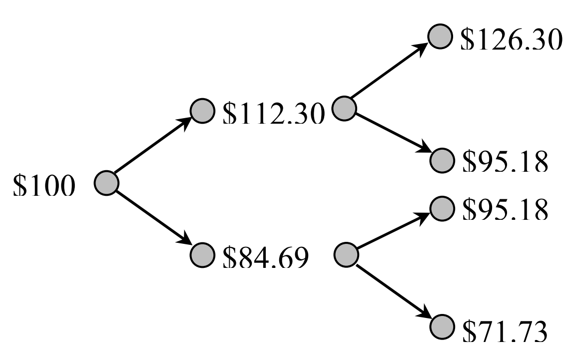
$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.20\sqrt{0.5}} = 1.1519 \qquad \text{[Eq. 4.23]}$$

$$d = e^{-\sigma\sqrt{\Delta t}} = e^{-0.20\sqrt{0.5}} = 0.8681 \qquad \text{[Eq. 4.24]}$$

$$q = \frac{(1 + r\Delta t - d)}{(u - d)} = \frac{(1 + (0.025 \bullet 0.50) - 0.8681)}{(1.1519 - 0.8681)} = 0.51 \qquad \text{[Eq. 4.25]}$$

$$1 - q = 0.49 \qquad \text{[Eq. 4.26]}$$

Figure 4.11: Binomial Decision Tree Example Results



At the end of the one year time period, the outcomes for the initial investment opportunity range from a high of \$126.30 (with a 26% probability of occurrence, i.e. 0.51 • 0.51) to a low of \$71.73 (with a 24% probability of occurrence, i.e. 0.49 • 0.49), with a most probable value of \$95.18 with a 50% probability of occurrence (i.e. 0.51 • 0.49 • 2). In this instance, where the same r_f, σ, u, and d are maintained for both periods, the *Sud* and *Sdu* values are equal. This mirrors the results that would be obtained using a recombining decision lattice. The power of the decision tree approach is that it allows reconsideration of these values at successive decision nodes, capturing the path dependency.

4.4 Float Valuation within Network Schedule Systems

The decisions or options facing construction projects, the realization of uncertainty, and the correlation of schedule flexibility to float, manifest in the decision to expend (or not to expend) float. To translate real options, capital budgeting methods, and prescient value of waiting for further information before acting on a decision relative to network schedule systems and determine a valuation methodology for trading float among the critical participants, this research presents calculations, analysis, and conclusions by way of an exemplar.

4.4.1 Research Expectation

The overarching expectation is that this research depicts a method for the measurement, allocation and pricing of risk within network schedule systems as represented by the consumption of float that addresses the unique treatment and understanding of total float. Float is a vanishing commodity that it is generally consumed on a first-come, first-serve basis and is not owned by any single entity (owner or contractor) or participant (subcontractors). More importantly this segment of the research triplet seeks a method that defines an equitable means for valuing (pricing) float for exchange among the participants most in need of its flexibility: critical network participants (those on the critical path who by definition have no float available).

4.4.2 Exemplar Development

4.4.2.1 Exemplar Foundation Elements

Reviewing the body of literature and analogous research, a simple network schedule used to depict network complexity and differing time calculation methods (Lucko 2005) is expanded to depict a project whose attributes and performance can easily, but accurately, portray the concepts under development and lend credibility to its analysis, conclusion(s), and extension.

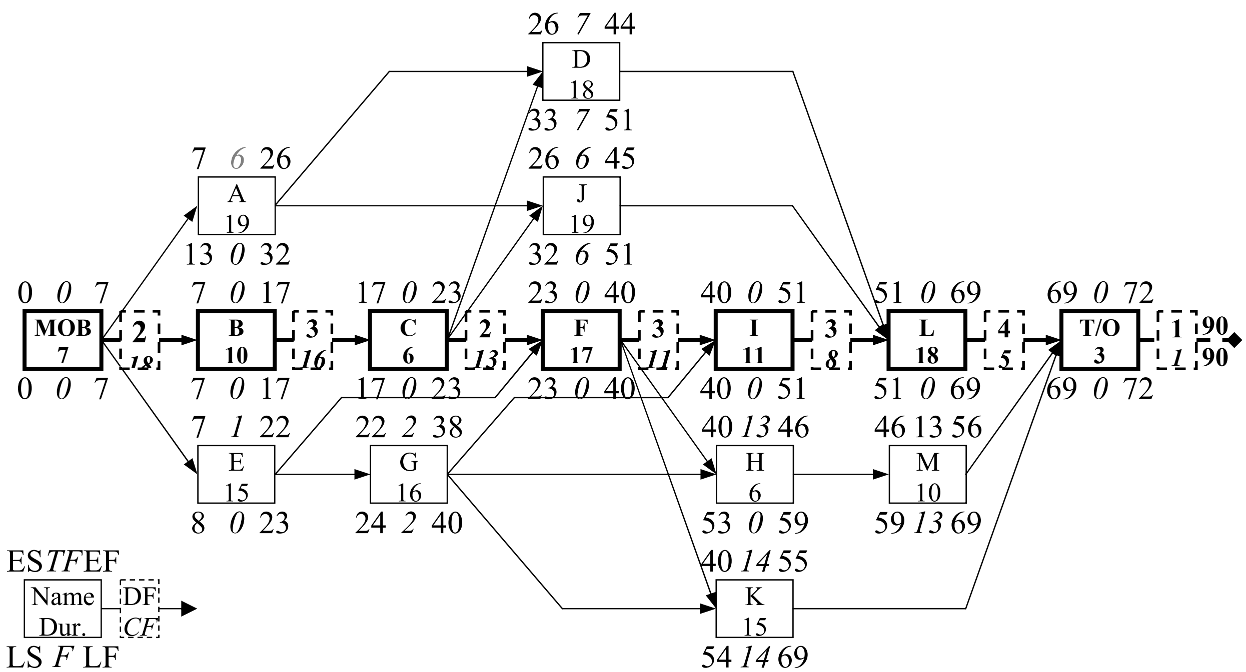
Table 4.19: Exemplar Inputs – Cost and Schedule Activity List with CPM Calculation Results and Distributed Float

Activity	Cost	Duration (days)	Successor	Early Start (ES)	Late Start (LS)	Early Finish (EF)	Late Finish (LF)	Total Float (TF)	Distri-buted Float
Mob.	**$50,000**	**7**	**A, B, E**	**0**	**0**	**7**	**7**	**0**	**2**
A	$285,000	19	D, I, J	7	13	26	32	6	
B	**$145,000**	**10**	**C**	**7**	**7**	**17**	**17**	**0**	**3**
C	**$25,000**	**6**	**D, F, J**	**17**	**17**	**23**	**23**	**0**	**2**
D	$210,000	18	L	26	33	44	51	7	
E	$150,000	15	F, G	7	8	22	23	1	
F	**$195,000**	**17**	**H, I, K**	**23**	**23**	**40**	**40**	**0**	**3**
G	$200,000	16	H, I, K	22	24	38	40	2	
H	$100,000	6	M	40	53	46	59	13	
I	**$110,000**	**11**	**L**	**40**	**40**	**51**	**51**	**0**	**3**
J	$250,000	19	L	26	32	45	51	6	
K	$255,000	15	T/O	40	54	55	69	14	
L	**$310,000**	**18**	**T/O**	**51**	**51**	**69**	**69**	**0**	**4**
M	$190,000	10	T/O	46	59	56	69	13	
Turn Over	**$25,000**	**3**	**N/A**	**69**	**69**	**72**	**72**	**0**	**1**
Total	$2,500,000	72 days							

Boldface activities are on the critical path.
Contractual duration: 90 days
Adopted from Lucko (2005) [costs added]
Distributed float from Chapter 3, Table 3.6b

Table 4.19 summarizes the critical path elements found in the literature, and Figure 4.12 graphically portrays the network logic, to which activity costs have been added representing those that could be expected of a small to mid-sized project of any type (construction-based projects not limiting) along with the 18 days of distributed and contract float derived within Chapter 3.

Figure 4.12: Exemplar CPM Network Schedule Diagram
Including Distributed and Contract Floats (Thompson and Lucko 2011)



In addition, to further the exemplar to a real options approach to float valuation, several other criteria must be defined, extrapolations made, and assumptions identified. Much like financial options, trading float is two-sided. It requires critical activities on both sides of the transaction: a buyer (an activity in need of additional time to complete their work), and a seller (an activity with distributed float that is no longer needed. Time may have overcome the activity and it is complete, or it has been determined that the probability for exceeding planned durations is low or nonexistent. The determination of real option variables may take different forms depending upon which side of the transaction is being evaluated. To accommodate this, the following discussion of the real options variables necessary for this exemplar is warranted.

4.4.2.2 Present Value of Investment / Time Value of Money

Float valuation opportunities are considered one-off occurrences, for which discounting forward in a NPV manner is negligible and may not be considered. As it applied to this exemplar, time extensions are likely to be in keeping with and proportional to the overall duration. Considering the 90-day contractual completion requirement, time extensions for which float valuation applies represent a fraction of the overall duration, e.g. less than 30%. Assuming a risk-free rate r_f at 5%, discounting a 30-day extension represents a reduction in value of less than one-half of one percent (actually 1/12 of 5%), a reduction of less than 50 cents per \$100 and \$4,149.38 on \$1 million.

4.4.2.3 Investment Cost

Buyer Side: The present value of the investment for an activity needing float (one where a schedule constraints will be exceeded) was notionally determined in de la Garza et al. (1991) *Total Float Traded as Commodity* as the difference between activity late finish costs (LFC) and early finish costs (EFC). However, the primary difference between late finish costs and the cost of modifying operations to meet the as-planned duration, i.e. schedule acceleration, is the extended overhead and general conditions costs.

Simply put, it is assumed that the cost to increase ongoing operations to meet the as-planned completion directly correspond to those that would be experienced beyond the as-planned duration. The only difference is the extended overhead / general conditions costs necessary to continue operations.

Seller Side: The de la Garza et al. formula cannot be applied to the seller side. The LFC costs do not exist because activity duration has been maintained. There is no need to expend additional cost, as work was completed on schedule. Rather, the potential investment costs for activities operating within duration constraints can be projected in standard terms as the daily general conditions cost or overhead of the operation. This is ubiquitously accepted / applied across the construction industry as the cost associated with contractual time extensions and varies greatly by specific activity and/or subcontractor. They are consistent with those recognized on the buyer side.

4.4.2.4 Time Period

With respect to the calculation of u and d, Eq. 4.23 and Eq. 4.24, the time period, Δt, is set at a default value of 1. This is predicated upon ignoring the discounting of the investment cost. This has the potential to impact the calculation of q by effectively removing the middle term, $r\Delta t$ (when coupled with the elimination of the interest rate ($r_f = 0$) and the time value of money). This is not important, as back-calculating the probabilities is a non-starter given the need for float. The probability for a schedule delay has already materialized (q = 1), rendering the probabilistic determination moot.

4.4.2.5 Uncertainty and/or Volatility

Buyer Side: Volatility on the buyer side of a float transaction connotes a heightened level of uncertainty. The need to acquire float directly correlates to increased volatility. Absent heightened schedule volatility, there would be no need to acquire float. Accordingly, the measure of uncertainty, σ, with respect to the acquisition of distributed float should be in the upper portion of the 25% to 50% range.

Seller Side: Uncertainty on the seller side of a float transaction is low. A prerequisite of the sale of float is certainty with respect to activity duration. Accordingly, the measure of uncertainty, σ, with respect to the sale of distributed float should be in the lower portion of the 25% to 50% range. However, another approach to the measure of uncertainty on the sale side of the transaction is dependent upon the period in which the transaction occurs. If the activity in the position to sell float has completed their work, the previously stated lower end value for σ holds, as schedule risk no longer remains. Conversely, when a distributed float transaction is positioned prior to seller activity completion, the measure of uncertainty must account for activity-specific schedule risk (the probability that the activity will need the float posited for exchange). In this event, a more appropriate extension of σ is the probability that the seller's activity will not finish within the as-planned duration, e.g. a 90% probability for as-planned completion yields a potential σ = 10% for needing float.

σ_c *vs.* σ_p*:* Given the need to differentiate the values for uncertainty and/or volatility from across sell side and buy side activities, σ_c will be used to define the buy side uncertainty associated with a call option; and σ_p will be used to define the sell side uncertainty associated with a put option. Simply, σ_c is defined as the overall project-specific uncertainty and should remain consistent across all activities as it addresses the collective or macro concerns and specifics of the larger project. σ_c is defined as the activity specific potential to need float, as it addresses the individual activity (specific subcontractor) ability to meet the as-planned duration along with consideration for the respective position in the sequence of activities, i.e. activities later in the schedule face greater uncertainty for beginning and completing their respective work as planned.

These real option components and their applications to the valuation of tradable float are identified in Table 4.20. Table 4.21 extends these elements to the critical activities of the exemplar – those with distributed float available for sale.

Table 4.20: Extended Exemplar Inputs – Real Options Relative Elements

Option Input	Variable	Float Correlation		Exemplar Value
Present Value of Investment	S	N/A		Investment Cost
Investment Cost	K	Daily Overhead / General Conditions Costs		Unique to Each Activity
Time Period	Δt	Individual Decision Occurrence (a onetime opportunity)		1
Value of Money Over Time	r_f	N/A		0
Volatility / Uncertainty	σ	Buyer Side (σ_c)	Schedule Volatility	Unique to Each Activity (0.25 to 0.50)
		Seller Side (σ_p)	Probability for Needing Float	Unique to Each Activity (0.00 to 1.00)

4.4.3 Pricing Distributed Float via the Real Option Binomial Decision Tree Method

4.4.3.1 Calculation / Equation Derivation

Calculating the value of float for exchange between critical activities by the adaptation of the binomial decision tree variant of real options theory as defined by Cox et al. is accomplished by extension of the calculation represented by Eq. 27 and is broken down into its component parts in Figure 4.13.

Figure 4.13: Analytic Breakdown of Call Value of Float Equation

NPV of underlying asset

$$C(X,T) = (S - K) \bullet u \;/\; (1 - r_f)^{\Delta t} \qquad \text{[Eq. 4.27]}$$

Volatility of potential decision = $e^{\sigma\sqrt{\Delta t}}$

Discount factor

Applying the correlations presented in Table 4.20, Eq. 4.27 simplifies to call value C(X) and a put value P(X) equations, Eq. 4.28 and Eq. 4.29 respectively, and requires the investment cost, the respective buy side and sell side volatilities, and the ability to consider the time value of money.

$$C(X) = \frac{K \bullet e^{\sigma_c}}{(1 - r_f)^T} \qquad \text{[Eq. 4.28]}$$

$$P(X) = \frac{K \bullet e^{\sigma_p}}{(1 - r_f)^T} \qquad \text{[Eq. 4.29]}$$

Note: For projects of short duration, those less than one year *($T < 1.0$)*, the inclusion a discount factor may not be appropriate when pricing put *P(X)* and/or call *C(X)*. This can be accommodated by setting $T = 0$, such that the value of $(1 - r_f)^T$ equals 1.

4.4.3.2 Distributed Float Valuation

In addition to the exemplar inputs relative to distributed float value, the assigned volatilities and overhead / general conditions costs, Table 4.21 contains the resulting buy side and sell side valuations for distributed float.

Table 4.21: Exemplar Critical Activity Inputs and Resulting Real Option Values

Activity	Investment Cost (*K*)	Uncertainty (σ)		Float Values			
				Buy Side		Sell Side	
		Buy Side (σ_c)	Sell Side (σ_p)	u_c	Call Value *C(X)*	u_p	Put Value *P(X)*
Mob.	$1,000	0.40	0.10	1.4918	$1,492	1.1052	$1,105
B	$4,500	0.40	0.15	1.4918	$6,713	1.1618	$5,228
C	$625	0.40	0.20	1.4918	$932	1.2214	$763
F	**$2,000**	**0.40**	**0.20**	**1.4918**	**$2,984**	**1.2214**	**$2,443**
I	$1,500	0.40	0.30	1.4918	$2,238	1.3499	$2,025
L	$3,500	0.40	0.45	1.4918	$5,221	1.5683	$5,489
Turn Over	$1,000	0.40	0.25	1.4918	$1,492	1.2840	$1,284

Costs in dollars per day

Tracing a single activity valuation calculation from beginning to end, the binomial decision tree valuation variant of real options theory for critical activity *F* (highlighted in **bold** font within Table 4.21) requires the following sequence: (1) determine u_c and u_p , and (2) multiply by *K* (the time value of money is being ignored, as the exemplar duration is less than one year, setting T = 0).

Call Value Calculation for Activity *F*, *C(F)*:

(1) $u_c = e^{\sigma_c \sqrt{\Delta t}} = e^{0.40 \cdot \sqrt{1}} = e^{0.40} = 1.4918$ [Eq. 4.30]

(2) $C(F) = u_c \cdot K = 1.49 \cdot \$2{,}000 = \$\ 2{,}984$ [Eq. 4.31]

Put Value Calculation for Activity *F*, *P(F)*:

(1) $u_p = e^{\sigma_p \sqrt{\Delta t}} = e^{0.20 \cdot \sqrt{1}} = e^{0.20} = 1.2214$ [Eq. 4.32]

(2) $P(F) = u_p \cdot K = 1.22 \cdot \$2{,}000 = \$\,2{,}443$ [Eq. 4.33]

4.4.3.3 Calculation Analysis

Analyzing the contents of Table 4.21, a correlation between the uncertainty element, the resulting *u* value, and the increase in the value of float for acquisition or disposition becomes obvious. The greater the schedule volatility (represented by σ_c) and the more probable the need for distributed float in the future (represented by σ_p), the more valuable float becomes to critical activities, and the higher the premium required to enter a transaction. Schedule volatility (σ_c) remains independent of relative position in the schedule, i.e. σ_c is consistent across all activities and is not time dependent. Activity uncertainty (σ_p) is specific to individual activities, i.e. σ_p is in part a function of time whereby the probability to finish as planned remains dependent on predecessor activity, thereby increasing volatility. Activity uncertainty (σ_p) is an increasing function with relative activity sequence / position.

4.4.3.4 Distributed Float Transaction

Focusing on the resulting *C(K)* and *P(K)* values for critical activities and the need to enter into a float transaction, identification of a potential schedule scenario becomes necessary. Extending the exemplar to the need to trade distributed float, consider the following scenario for the completion of activity *F*: 32 days have elapsed with activities *Mob*, *B*, and *C* having finished as planned; activity *F* is nine days into its work and is expecting that the planned duration of 17 days will be exceeded by ten days.

This necessitates the expenditure of the three days of distributed float assigned to activity *F* plus the need for an additional seven days of float to keep the project on track and meet the 90 day contractual completion schedule. Activity *F* needs to acquire distributed float from other critical activities within the network schedule system that have either completed their activities and no longer need float (but retain ownership of it), or have sufficient confidence that their work will be completed as planned (characterized by a sufficiently low σ_p).

Table 4.22: Ascending Real Option
Call Values *C(X)*and Put Values *(P(X)*

Activity	Call Value *C(X)*	Put Value *P(X)*	Distributed Float
C	$932	$763	2
Mob.	$1,492	$1,105	2
Turn Over	$1,492	$1,284	1
I	$2,238	$2,025	3
F	**$2,984**	**$2,443**	**3**
L	$5,221	$5,228	4
B	$6,713	$5,489	3

Costs in dollars per day

Based upon the sorted values depicted in Table 4.22, the call value *C(X)* for activity *F* is greater than the put values *P(X)* for activities *C*, *Mob*, *Turn Over*, and *I*. With the exception of activities *I* and *Turn Over*, the work of the aforementioned activities has been completed so that a float transaction can be entered without consideration for activity uncertainty (σ_p). Accordingly, activity *F* can exact a float purchase of four days distributed float below its *C(X)* value of $2,984 per day and the remaining three days of needed float from activity *B* at $5,489 should no other opportunities for a transaction materialize.

To avoid costs above the call value for distributed float, activity *F* should seek to acquire the last three days of needed float from activities *I* and *Turn Over*, as their respective put values are below the call value of activity *F*. Entering a float transaction with activity *F* is dependent upon sufficient confidence in the ability to complete their work within the as-planned schedule constraints. That is, enough time must have elapsed in the overall schedule that the initial σ_p for activities *I* and *Turn Over* can be reevaluated and reduced such that the 0.30 and 0.25 values approach zero.

4.4.3.5 Real Option Considerations

Returning to the foundation of float valuation real options analysis, trading float based upon the values identified in Table 4.21 and the subsequent analysis departs from a binomial valuation effort and shifts to a decision process that may be as simple as the those depicted in Figures 4.6 and 4.10 (but may have nested options) as previously defined: (1) expend float where available to mitigate the schedule impact – including the acquisition of distributed float, or (2) accept the schedule delay and the associated effects.

Translating the binomial decision tree for the expenditure of float into tabular format, the decisions and sub-variants (nested options) are depicted in Table 4.23. Specific costs can be determined for each nested option and any associated variant – inclusive or exclusive of the time value of money, to aid in determining the appropriate course of action.

The exemplar decision for activity *F* fits within nested option variant Sud_1 "Sufficient Distributed Float Available for Acquisition within *C(X)* Value." However, given the indeterminate status of the distributed float of activity *I*, the real option decision could escalate to Sud_2, where there is insufficient distributed float available within the call value.

Table 4.23: Binomial Decision Table for the Expenditure of Distributed Float – the ‘Need to Overcome a Schedule Challenge (*S*)’

<table>
<tr><th>Binomial Option</th><th colspan="2">Nested Option and Variant</th><th>Decision</th></tr>
<tr><td rowspan="5">Expend Distributed Float ‘DF’ (Su)</td><td colspan="2">Float Needed Less Than (<) Owned DF (Suu)</td><td>Expend Activity Owned DF (DF remains Available for Trade)</td></tr>
<tr><td rowspan="4">Float Needed Greater Than (>) Owned DF (Sud)</td><td>Sufficient DF Available for Acquisition within C(X) Value (Sud_1)</td><td>Incorporate Above Decision and Acquire Available DF within C(X) Value</td></tr>
<tr><td rowspan="2">Insufficient DF Available for Acquisition within C(X) Value (Sud_2)</td><td>Incorporate Above Decision and Acquire Available DF above C(X) Value, or</td></tr>
<tr><td>Incorporate Above Decision (Acquire Available DF within C(X) Value) and Implement Acceleration Methods</td></tr>
<tr><td>Insufficient DF Available to Fulfill Need (Sud_3)</td><td>Incorporate Above Decision and Include Acceleration Methods for Remaining Need</td></tr>
<tr><td rowspan="3">Do Not Expend Distributed Float ‘DF’ (Sd)</td><td colspan="2">Schedule Extension Available without Consequence (Sdu)</td><td>Make No Schedule Accommodations</td></tr>
<tr><td rowspan="2">Schedule Extension Available with Consequence (Sdd)</td><td>Consequence / Effects of Schedule Extension Acceptable (Sdd_1)</td><td>Incorporate Above Decision</td></tr>
<tr><td>Consequence / Effects of Schedule Extension Not Acceptable (Sdd_2)</td><td>Develop Acceleration Methods to Mitigate Impact of Consequences</td></tr>
</table>

4.5 Application

4.5.1 Construction Duration Variability

Construction projects, and in particular critical infrastructure projects, are notorious for time and cost overruns (Creedy et al. 2010, Shane et al. 2009, Kim 2007, Georgy et al. 2000). The overruns are in part due to a “deficiency in managing the scope, time, quality, cost, [and] productivity” (Jergeas and Ruwanpura 2010, p.40). Depending on project type, schedule variation ranges from early finishes to far exceeding planned durations. An international study of over 200 building projects portrayed that approximately one-third finished on or ahead of planned duration (Acharya et al. 2006), with 20% exceeding their planned duration by more than 50% (Table 4.15 depicts the full schedule variability findings of Acharya et al.).

Project management is doubly challenged as “poor scheduling and control” are contributing causes for these overruns (Akpan and Igwe 2001, p.367, Jergeas and Ruwanpura 2010), and then managers remain challenged to mitigate their combined impact. The introduction of a float valuation method represents a vehicle to overcome these challenges and maximize the flexibility available to critical activities. To demonstrate this, duration variation is applied to the exemplar via Monte Carlo simulation, the results of which will form the data population for statistical analysis to support the hypothesis of the exemplar.

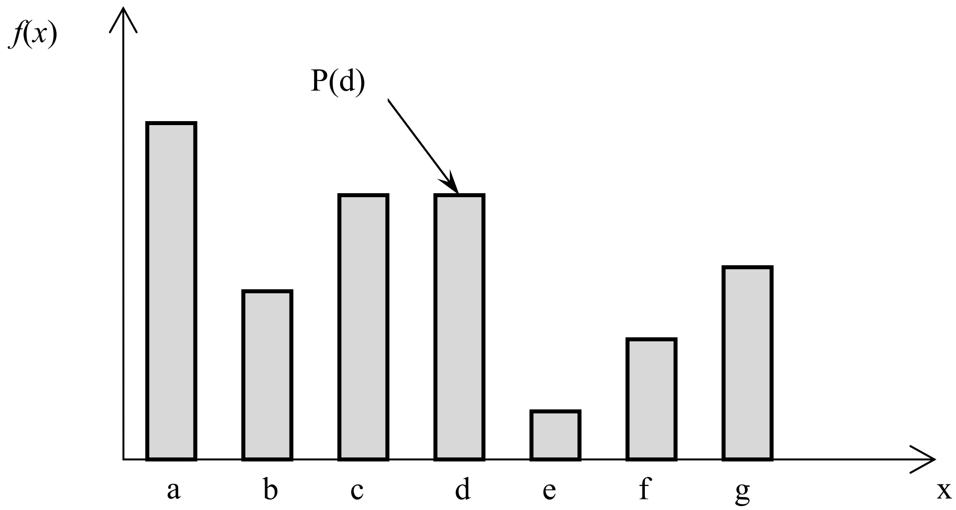
4.5.2 Probability Distribution Functions

Modeling network schedule systems within the construction industry is dependent upon selection of the appropriate probability distribution function (PDF), the mathematical function that describes the probability of a random variable taking certain values. Considerable research has been undertaken to determine the appropriate PDF to imitate the uncertainty and variability of schedule durations and/or costs. Arízaga (2007) determined that several PDFs are suitable for use in construction industry modeling. They include the normal and lognormal distributions (Touran 1997), the beta distribution (Touran 1997, Fente et al. 2000, Maio et al. 2000, Schexnayder et al. 2005), and the triangular distribution (Back et al. 2000, Arízaga 2007). Wilson et al. (1982) studied the use of beta versus triangular distributions on NASA ground operations, concluding that there were not significant differences in the simulation outputs. PDFs fit two molds, discrete and continuous.

4.5.2.1 Discrete Probability Distribution Functions

A discrete PDF is a random variable representation that by definition can assume only a finite or 'countably infinite' number of values. Its values are distributed over rational numbers at isolated points; they are integers (Everitt 2006). Some of the most common discrete PDFs used in statistical modeling and simulation are the Poisson and Bernoulli distributions, the binomial distribution, the geometric distribution, and the discrete uniform PDF (common to computer programming) that makes random equal-probability choices between a number of specifically-defined choices. When graphically depicted, discrete distributions are discontinuous; its values are isolated (Figure 4.14).

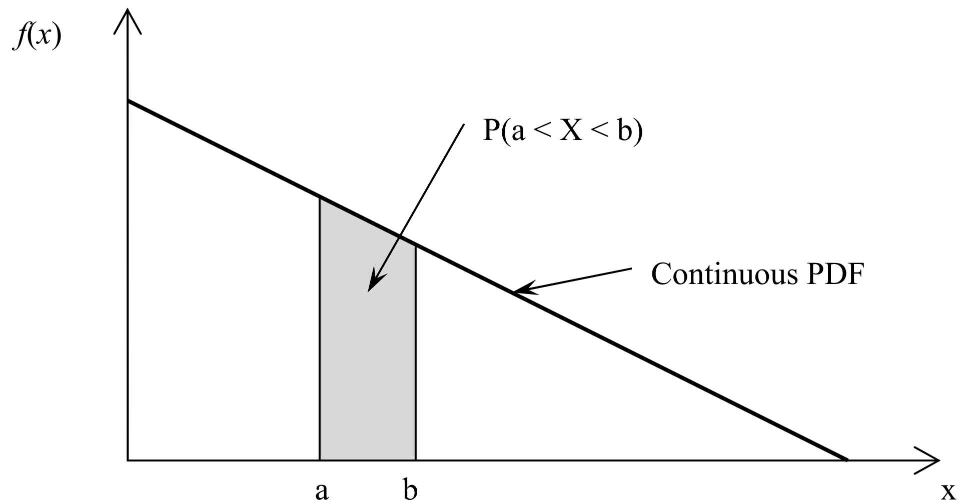
Figure 4.14: Graphic Depiction of Discrete Probability Distribution Function



4.5.2.2 Continuous Probability Distribution Functions

A continuous PDF is a random variable representation that by definition can assume only an 'absolutely continuous' number of values. It is the opposite of a discrete PDF. A continuous PDF is associated with and must have a density function. That is, a continuous PDF is represented by a formula where the probability of returning a value between two limits is equal to the area under the line or curve representing the formula (its density or mass) and the probability of returning a specific value is zero (Everitt 2006). Examples of continuous PDFs are the normal and lognormal distributions, the uniform distribution, and the chi-squared distribution. When graphically depicted, the continuous distributions are unbroken; specific values cannot be determined (Figure 4.15).

Figure 4.15: Graphic Depiction of Continuous Probability Distribution Function



4.5.3 The Triangular Probability Distribution Function

Modeling risk implies a stochastic (probabilistic) process whose quantification is uniquely adaptable to modeling by the application of a PDF to each element in a cost estimate or schedule (Raymond 1999). When applying Monte Carlo techniques:

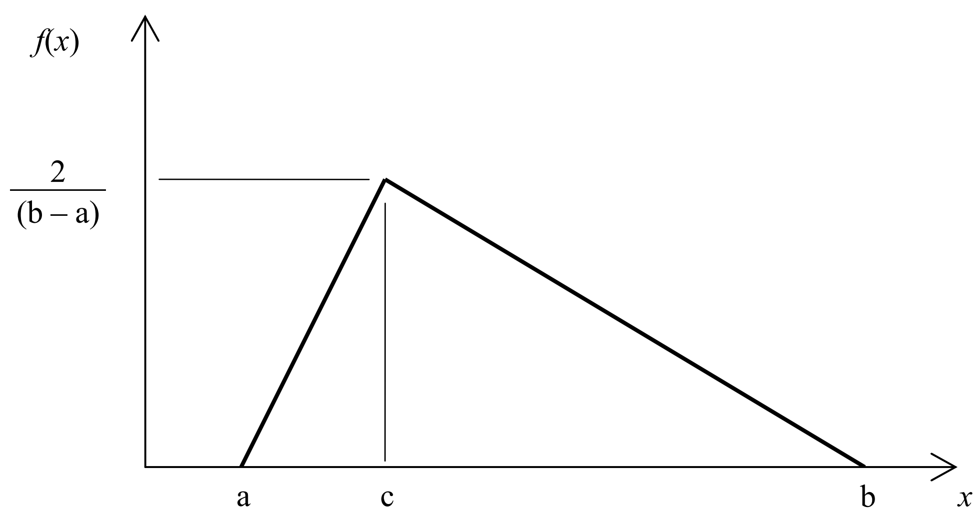
> A simple triangular distribution is a reasonable PDF for describing risk or the uncertainty for a cost element or task duration estimate. Its structure is based on the minimum possible cost and duration (plan best case), the most likely cost and duration (budget most likely) and the maximum possible cost and duration (project worst case)…The parameters are simple, intuitively easy to comprehend, and amenable to a mathematical formulation comparable with cost and schedule models and fast Monte Carlo analysis. Other more complex distributions could be used such as the Beta or Weibull, but little if anything is gained, and the intuitive simplicity of the triangular distribution is lost (Raymond 1999, p.148).

4.5.3.1 Particulars of the Triangular PDF

The triangular probability distribution is a continuous function and is defined by three points per Eq. 4.34 and Figure 4.16: (1) the minimum value *a*, (2) the most likely value or statistical mode *c*, and (3) the maximum value *b*. The direction of the skew of the triangular distribution is set by the size of the most likely value relative to the minimum and the maximum. It is perhaps the most readily understandable and pragmatic distribution for basic risk modeling and has a number of desirable properties including a simple set of parameters, the use of a modal value (i.e. the most likely case), and a deterministic probability distribution generated by range of possible values.

$$f(x|a,b,c)=\begin{cases} 0 & x < a \\ \dfrac{2(x-a)}{(b-a)(c-a)} & a \le x \le c \\ \dfrac{2}{(b-a)} & x = c \\ \dfrac{2(b-x)}{(b-a)(b-c)} & c < x \le b \\ 0 & x > b \end{cases} \qquad \text{[Eq. 4.34]}$$

Figure 4.16: Graphical Representation of Triangular Distribution



Conversely, the triangular distribution has two significant disadvantages. First, when the parameters result in a skewed distribution where the area of the triangle being predominantly on one side of the mode value, there may be an over-emphasis of the outcomes in the direction of the skew. Second, the distribution is bounded on both sides, whereas many processes are only bounded on one side but may lack real-life constraints and remain unbounded on the other side.

Beyond selection of the triangular PDF, additional constraints are required to implement a Monte Carlo simulation of the exemplar schedule network. Of primary concern are the *a* and *b* components required for the distribution, the lower and upper bounds of the distribution respectively, and the mode value *c*, herein considered the as-planned duration. The relationship between these components is demonstrated by the probability expectation for the occurrence of *c*, the mode value, defined by Equation 4.35.

$$P(x|c):\ \frac{2}{(b-a)} \qquad \text{[Eq. 4.35]}$$

4.5.3.2 Application of the Triangular PDF

The triangular PDF is a continuous distribution whose application to network schedule systems requires modification. Calculation of the individual PDF bounds (the range of the function, *a* and *b*), requires rounding as does the resulting variable value (the activity duration), because partial days are anathema to network schedule systems. The triangular PDF returns a continuous value, such as 4.1367348, when calculating the area under the curve (as represented within Figure 4.16), not an integer representing a whole day. To accommodate this, such results will be rounded to integers to appropriately represent whole days (by necessity from accepted scheduling convention and software limitations) by the using numeric (decimal) rounding rules.

To arrive at the baseline PDF to be used in the Monte Carlo Simulation, activity durations were varied as a percent function of the as-planned duration and analyzed against the idealized one-third – two-thirds Acharya et al. (2006) study distribution for schedule duration distribution versus the 72-day expected duration (exclusive if the 18 days of distributed float). To calibrate the model, that is to set the PDF for activity level performance, the individual as-planned durations were translated into a triangular PDF by multiplying the as-planned duration by one-plus or one-minus a percentage for the *b* and *a* PDF elements respectively (e.g. the 90% / 125% activity duration simulation results per Table 4.24 – Array A). They were created by multiplying each as-planned duration by 1-90% and 1+125% for the lower and upper durations (90% was selected as the lowest duration, as all activities require time; the 90% reduction defaults to minimum 1 day duration). The resulting durations were rounded to whole days.

This process was repeated at varying incremental percentages until the simulation results converged near the desired duration distribution and then evaluated at finer granularity to determine the best fit percentages. The Excel table used to generate the PDF values and populate the duration values for each activity data and calculation box used in the ranges necessary to calibrate the simulation model is depicted in Table 4.24, Array A, where *a* = minimum value, *b* = maximum value, and *c* = most likely value.

Table 4.24: Monte Carlo Simulation Activity Duration PDF Generator

Array A – Best Fit Values

Triangular PDF			
Percent < APD			90%
Percent > APD			125.0%
ID	**a**	**c**	**b**
Mob	1	*7*	16
A	2	*19*	43
B	1	*10*	23
C	1	*6*	14
D	2	*18*	41
E	2	*15*	34
F	2	*17*	38
G	2	*16*	36
H	1	*6*	14
I	1	*11*	25
J	2	*19*	43
K	2	*15*	34
L	2	*18*	41
M	1	*10*	23
T/O	1	*3*	7

APD = As-Planned Duration

Array B – Intuitive Values

Triangular PDF			
Intuitive Values			
ID	**a**	**c**	**b**
Mob	3	*7*	7
A	14	*19*	21
B	6	*10*	18
C	3	*6*	9
D	13	*18*	29
E	12	*15*	24
F	10	*17*	26
G	11	*16*	26
H	3	*6*	11
I	5	*11*	17
J	13	*19*	30
K	11	*15*	24
L	15	*18*	18
M	8	*10*	14
T/O	2	*3*	5

4.5.4 An Intuitive Approach to Activity Durations

Similar to the calibrated distribution version, an intuition-based approach to the activity duration range is offered. In this version, the activity durations are based upon experience and intuition to set the activity level duration variation from that planned. To arrive at the a and b components of the intuitive triangular PDF, elements such as sequence in the schedule, the level of parallel activity, comparative duration, and the potential for acceleration (as depicted in Table 4.25) were considered. The skew and potential for acceleration are dependent upon the activity characteristics as depicted in the exemplar CPM network diagram (Figure 4.16).

Table 4.25: Intuition-Based Simulation Critical Activity PDF Values and Characterizations

Probability Distribution Function Values					Skew	Position	Parallel Activity	Duration	Probability for Success	Acceleration Potential
ID	a	c	b	P(x|c)						
Mob	3	7	7	0.500	Left	Early	Low	Short	High	Yes
A	14	19	21	0.286	Left	Early	High	Long	Medium	Yes
B	6	10	18	0.167	Right	Early	High	Medium	High	No
C	3	6	9	0.333	None	Early	High	Short	High	No
D	13	18	29	0.125	Right	Middle	High	Long	Low	No
E	12	15	24	0.167	Right	Middle	High	Long	Medium	No
F	10	17	26	0.125	Right	Middle	High	Long	Medium	No
G	11	16	26	0.133	Right	Middle	Low	Long	High	No
H	3	6	11	0.250	Right	Middle	High	Short	Medium	No
I	5	11	17	0.167	None	Middle	High	Long	Low	Yes
J	13	19	30	0.118	Right	Middle	High	Long	Low	No
K	11	15	24	0.154	None	Middle	High	Long	Medium	Yes
L	15	18	18	0.667	Left	Later	Low	Short	High	Yes
M	8	10	14	0.333	Right	Later	Low	Medium	High	No
T/O	2	3	5	0.667	Right	Later	Low	Short	High	No

4.5.5 Monte Carlo Simulation

To provide further insight into the interactions of the exemplar schedule participants under the conditions of uncertainty and variability, a Monte Carlo simulation (or simulation) will be performed. Absent a vast array of network schedules with common participants (activity ownership) against which to compare as-planned versus actual durations and calculated float values via a real option analysis, a method to approximate a cohort of projects (to simulate the variety of results likely for the exemplar) is to apply PDF constraints to the exemplar schedule for which multiple iterations are then produced and analyzed.

4.5.6 Statistical Bootstrapping through Monte Carlo Simulation

Statistical bootstrapping is useful when the sample size is insufficient for straightforward statistical inference. It is generally used to estimate the distribution of a statistic (e.g. the mean, variance, etc.) when normal theory is unavailable to help estimate the distribution. The statistical distribution herein is the resulting activity-level and overall project schedule durations. To bootstrap data, the "data-based simulation method for statistical inference" (Efron and Tibshirani 1993, p.5), is to create an initial population of data (herein the initial exemplar as-planned network schedule) and then through drawing and replacement (herein individual iterations of the simulation model), a bootstrapped population of data (herein the marketplace) is derived. "…[R]epeat this process a large number of times, say 1000 times, to obtain a 1000 bootstrap replica [of the population]" (Efron and Tibshirani 1993, p.5). "The particular goal of bootstrap theory is a computer-based implementation of basic statistical concepts" (Efron and Tibshirani 1993, p.6).

The purpose of Monte Carlo simulation is to bring variability and uncertainty to the activity durations within the exemplar network schedule system and gauge the performance of the activities forming the critical path with respect to that of the entire schedule system. The activity duration population generated by the simulation model will serve as the bootstrapped data for the calculation of call and put values for trading float by the critical activities of the schedule system (activities *Mob*, *B*, *C*, *F*, *I*, *L*, and *T/O*) using Equations 4.28 and 4.29. It is expected that this will provide insight into the validity of the float values drawn from the exemplar hypothetical values.

4.5.7 Monte Carlo Simulation Model Development

The Monte Carlo simulation model used to bootstrap a statistical population was developed using the @Risk™ (At-Risk) Risk Analysis and Simulation Add-In Program, Version 5.7 (September, 2010) for Microsoft® Excel (2007), from the Palisade Corporation, Ithaca, NY.

To create the working model representing the exemplar network schedule system, an activity data and calculation box representing the schedule components for each activity and the allocated distributed float was developed and positioned within the Excel worksheet in relative position to that depicted in the Exemplar CPM network diagram (Figure 4.12). Excel "MAX" statements set the predecessor-to-successor logic for the early start (ES) of each activity (with the maximum being the maximum schedule duration taken by the predecessor activities requiring completion before the successor may begin). Early finish (EF) duration is calculated by adding the duration of the subject activity to the early start. This is completed for all activities such that the schedule forward pass is complete. A backward pass is then completed using only "EQUAL" statements from activity to activity upon subtraction of the activity duration from the late finish (LF) to create the early finish (EF). A portion of the model is depicted in Figure 4.17. The activity and data and calculation box for activity *I* is deconstructed in Figure 4.18, including the @Risk PDF parameters and output expectations.

Figure 4.17: Excel-Based @Risk Simulation Model – Partial Segment

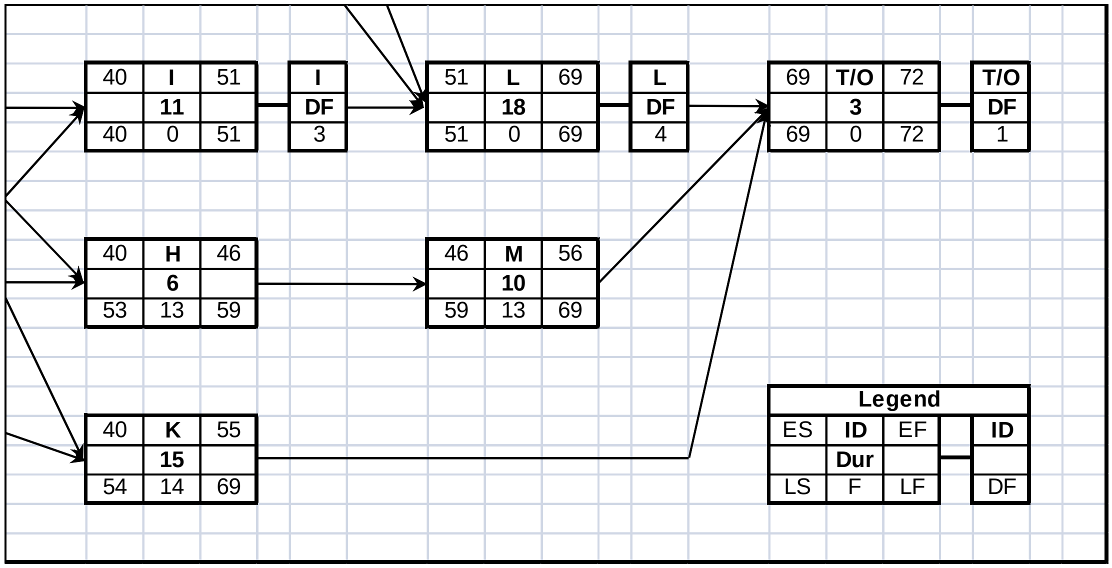
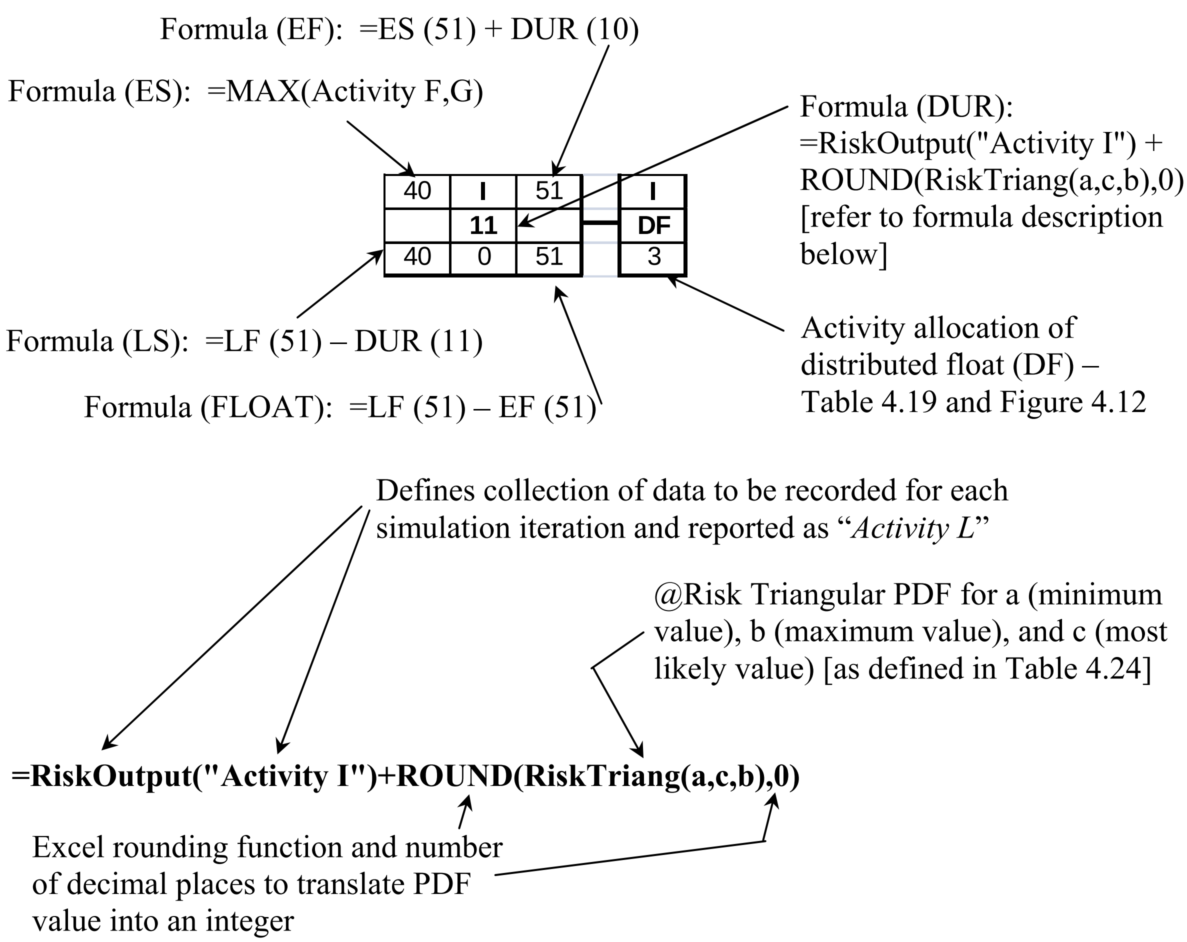


Figure 4.18: Data and Calculation Box Deconstruction – Activity *I*



4.5.8 Simulation Results

The results for the simulation depict the durations for the critical activities and the overall schedule. They depict the results from several PDF variations that were then blended to yield bootstrapped values for activity and project volatility and uncertainty, the buyer side σ_c (schedule volatility) and seller side σ_p (the probability for needing float). These values are used to validate hypothetical float values from the exemplar. The model variations selected, their targeted and resulting durations, and the justification for inclusion in the data population are summarized in Table 4.26. The variations selected for data generation and statistical blending are:

- 25% / 28.5% Variation: This mimics the Acharya et al. (2006) project completion percentages within the exemplar and serves as the baseline for modeling herein. It results in a 74-day overall network duration.
- 25% / 22% Variation: Calibrated to achieve the as-planned 72-day duration.
- The Intuitive Variation: Designed to approximate the manner in which a project manager would originally schedule and then manage the Exemplar network.

Table 4.26: Simulation Model Combined Calibration, Validation, and Justification

Model Calibration			**Model Validation / Justification**	
Less than APD*	**Greater than APD**	**Target Duration (days)**	**Average Duration (days)**	**Variation Intent**
25%	**28.5%**	N/A	73.86	Fulfills Acharya et al. 1/3 – 2/3 duration distribution
25%	**22%**	72	72.15	Achieves 72-day as-planned duration
Intuitive		N/A	74.89	Project Management Approach

* APD = As-Planned Duration

100,000 iterations per simulation model

Simulation results for critical activities and the total network duration across the model variations identified in Table 4.26 are summarized in Table 4.27, along with the relevant statistical derivations for on-time completion and variation from as-planned duration (its standard deviation σ).

Table 4.27: Simulation Model Results – Critical Activity and Network Durations

Activity	Duration (Days)		On Time
	As-Planned	Simulation	
25% / 28% Model Variation			
Mob	7	7.00	71.88%
B	10	10.33	58.33%
C	6	6.33	62.50%
F	17	17.33	55.00%
I	11	11.00	65.28%
L	18	18.33	55.00%
T/O	3	3.00	87.50%
Total	72	73.89	33.63%
25% / 22% Model Variation			
Mob	7	7.00	71.88%
B	10	10.00	71.88%
C	6	6.00	87.50%
F	17	17.00	61.72%
I	11	10.67	77.50%
L	18	18.00	61.72%
T/O	3	3.00	87.50%
Total	72	72.15	55.12%
Intuitive Variation			
Mob	7	5.69	100%
B	10	11.33	41.41%
C	6	6.00	65.28%
F	17	17.67	49.83%
I	11	11.00	57.99%
L	18	17.03	100%
T/O	3	3.33	62.50%
Total	72	74.89	30.55%

The data within Table 4.27 represent the comparison between the as-planned duration for the critical activities and the overall network schedule system. From the 100,000 iterations of each model variation, the average duration was calculated, as was the count for iterations finishing at or ahead of the as-planned duration. This is represented as the percent of on-time completion and its reciprocal, the percent of iterations delayed. The reciprocal value, when blended across the three variations, will serve as the values for σ_c (specific to each critical activity) and σ_p (for the overall network schedule), the buy side and sell side uncertainties used in determining the real option value of tradable float.

To arrive at a single value to use for uncertainty (σ_c and σ_p), the resulting reciprocal values for schedule completion, i.e. delayed finish percentages, will be averaged (blended). This further bootstraps data to represent the expected durations irrespective of specific modeling and/or scheduling approach employed. These values are presented in Table 4.28.

Table 4.28: Simulation Model Results – Blended Values

Activity	Duration (Days)		On Time	Delayed
	As-Planned	Simulation		
Mob	7	6.56	81.25%	18.75%
B	10	10.55	57.21%	42.79%
C	6	6.11	71.76%	28.24%
F	17	17.33	55.52%	44.48%
I	11	10.89	66.92%	33.08%
L	18	17.79	72.24%	27.76%
T/O	3	3.11	79.17%	20.83%
Total	72	73.64	39.77%	60.23%

4.5.9 Simulated Distributed Float Valuation and Analysis

Using the same process as for the exemplar to value float (per Table 4.21), values for distributed float for the call side and put side transactions based upon the uncertainty values garnered from the blended simulation results (Table 4.28) are summarized in Table 4.29.

Calculation inputs follow from the hypothetical exemplar values as follows:

- *Investment Cost (K):* The daily overhead / general conditions costs – Remain valid from initial exemplar calculations
- *Buy Side Uncertainty (σ_c):* Overall schedule volatility (the project dynamic between activities) – New value derived from the overall network probability to finish on time, equal to 0.60
- *Sell Side Uncertainty (σ_p):* Probability for needing float (the project dynamic within each activity) – New value derived from individual activity probability to not finish on time is unique to each critical activity

Table 4.29: Simulation Critical Activity Inputs and Resulting Real Option Values

Activity	Investment Cost (K)	Uncertainty (σ)		Float Values			
				Buy Side		Sell Side	
		Buy Side (σ_c)	Sell Side (σ_p)	u_c	Call Value *C(X)*	u_p	Put Value *P(X)*
Mob.	$1,000	0.60	0.19	1.8221	$1,822	1.2092	$1,209
B	$4,500	0.60	0.48	1.8221	$8,200	1.6161	$7,272
C	$625	0.60	0.23	1.8221	$1,139	1.2586	$787
F	$2,000	0.60	0.44	1.8221	$3,644	1.5527	$3,105
I	$1,500	0.60	0.33	1.8221	$2,733	1.3910	$2,086
L	$3,500	0.60	0.28	1.8221	$6,377	1.3231	$4,631
Turn Over	$1,000	0.60	0.33	1.8221	$1,822	1.3910	$1,391

Costs in dollars per day

Comparing the results from the initial hypothetical exemplar float values to that of the values obtained with the inputs emanating from the simulation, several differences become apparent:

- The value of σ_c for the buy side derived from simulation bootstrapping significantly exceeds that assumed by extension for other industries (0.40 by extension versus 0.60 from simulation). This uniformly increases the buy side float values by 22%.
- The values of σ_p for the sell side derived from simulation bootstrapping represent a tighter range that that assumed by relative schedule position (0.10 to 0.45 based on position versus 0.19 to 0.48 from simulation, a simulated range of 0.29 compared to an position-specific range of 0.35)

Comparative values for buy side (call value) and sell side (put value) are summarized in Table 4.30). The simulation-derived results produce greater volatility that originally anticipated.

Table 4.30: Simulation versus Hypothetical Float Real Option Values

Activity	Float Values					
	Buy Side Call Value *C(X)*			Sell Side Put Value *P(X)*		
	Exemplar Hypothetical	Simulation	Delta	Exemplar Hypothetical	Simulation	Delta
Mob.	$1,492	$1,822	22.12%	$1,105	$1,209	9.41%
B	$6,713	$8,200	22.15%	$5,228	$7,272	39.10%
C	$932	$1,139	22.21%	$763	$787	3.15%
F	$2,984	$3,644	22.12%	$2,443	$3,105	27.10%
I	$2,238	$2,733	22.12%	$2,025	$2,086	3.01%
L	$5,221	$6,377	22.14%	$5,489	$4,631	15.63%
Turn Over	$1,492	$1,822	22.12%	$1,284	$1,391	8.33%
Standard deviation	$2,165	$2,644	22.12%	$1,955	$2,324	18.87%

Costs in dollars per day

4.6 Conclusions

This research began with the real option extension of capital budgeting and finance and the premise that their precepts could be extended to network schedule systems and the expectation of a pricing methodology for the trading of float among a network schedule system's participants, its individual activities. Through the literature, real option theory was confirmed as an appropriate means for determining the individual activity specific values at which float could be bought and sold between critical activities. It also extended the binomial decision tree method to price float incorporating risk in the form of schedule volatility associated with the project as a whole (σ_c), as well as the activity specific uncertainty for meeting as-planned durations (σ_p).

Building upon the real option application demonstrated by exemplar, this research concludes that the valuation method for the recently characterized distributed float (DF) (Thompson and Lucko 2011) within critical path network systems can employ a real option decision process of binomial nature, to expend float or not to expend float, following the developed call and put values as determined by Eq. 4.28 and Eq. 4.29 respectively. Included within these values is the option to include the time value of money based upon the specific duration of the schedule at hand, and the Bernanke (1983) premise of waiting for additional information has inherent value. Some decisions, like the float decision represented herein, are irreversible (or prohibitively expensive to undo / overcome).

By definition an option, the future right of choice but not the obligation to act, provides a logical extension to the decision to trade float. The adaptation of real option theory to accommodate uncertainty (risk) within schedule systems and the application of proven financial and analytical concepts represents the final element in fulfillment of the de la Garza et al. (1991) premise of trading total float as a commodity: a pricing model.

In addition, this research brings forward the idea of using general conditions / overhead costs as the base of the investment cost component of a real option valuation. The difference between a late finish and modifying operations to meet the schedule is the extended overhead or general conditions costs. This accommodates one side of the real option transaction, the call value or need to acquire float. Similarly, when float is available for exchange, the put value or ability to forego float by sale, a corresponding investment cost is needed. This research concludes that the same overhead / general conditions costs should be employed, as this is the foundation by which added scope would be marked up.

This research concludes that for a real option approach to pricing float for exchange, the underlying asset is equal to the daily cost to operate, and it must be accompanied by appropriate values for uncertainty: both inherent to the schedule network, to its participants, and recognizing the assertion that waiting for more or better information also impacts the valuation process.

4.7 Future Research

This new approach to pricing float (i.e. risk) based upon previously vetted capital budgeting and financial analysis tools opens extended avenues for future research. As initially introduced, this research represents the final element in the three-part fulfillment of risk quantification, price and mitigation, allocation, and the development of a prediction method for where risk is likely to reside, is focused on the pricing component. As a valid risk pricing model has been crafted via real options theory, future investigation depicting analogous research and extending the aforementioned components of the *Total Float Traded as Commodity* notion of de la Garza et al. (1991) is warranted. In specific, future research resulting from this investigation should focus on the quantification of construction industry specific project-level and activity-level uncertainty and/or volatility, the σ_c and σ_p components of the value equation (Eqs. 4.28 and 4.29).

It is expected that the remaining elements in this three-part research endeavor will complete the components necessary for a float trading means will address risk's predominant location, magnitude, and subsequent allocation within network schedule systems, and in particular within construction project CPM networks. In similar form to the methods employed herein, subsequent research should engage concepts currently vetted, in existence, or in practice. It is expected that a working predictive modeling mechanism will result from the exploration and analysis of risk's residence in network schedule participants along with a market model for its exchange.

4.8 References

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CHAPTER 5

Summary and Conclusions

5.1 Summary

In environments where uncertainty remains an important factor, flexibility, adaptability, and the ability to delay decisions while awaiting additional and better information have inherent value. The same can be said for construction projects governed by network schedule systems, where uncertainty or risk is manifested through schedule delays. Project management needs to be prepared for different kinds of events across the life of a project and the ability to mitigate the uncertain and risky environment in which they operate. This dissertation proposes a comprehensive framework to identify, measure, allocate, and price risk within construction network schedule systems, along with the strategic flexibility for its integration as an integral part of project planning and execution.

It is founded upon the premise that previously developed theories within disciplines beyond the bounds of construction, engineering, and architecture by extension and adaptation can lend insight into the measurement, allocation and pricing of risk in network schedule systems. Financial portfolio theory and perfect competition were examined and extended as analogies for the identification and measurement of risk within schedule systems. Social decision-making and voting models, European models in particular, form the basis of risk allocation across network participants, while option theory and the irreversibility of decisions forms the basis for the monetization (pricing) of risk.

This research is based on the assertion that uncertainty, volatility, and risk are expressed in construction project schedule systems by deviation from the as-planned activity durations and are accommodated (mitigated in whole or in part) by the expenditure of float. Of primary importance is the recognition that not all float is equal, nor is float ownership evenly or completely distributed across a schedule system activity ownership.

Critical activities, those that form the critical path of a Critical Path Method (CPM) network schedule system (typically employed in construction projects) by definition have no float available to overcome the risks associated with their performance. It is this group upon which this research is focused and for which the framework developed provides a means to incorporate float along the critical path for exchange among its participants.

5.1.1 Background (Chapter 1)

The introductory section of this dissertation began with the depiction of the construction industry in the United States as seminal, an entity that generates nearly 4% of the U.S. GDP and employs over 5.6 million workers (Bureau of Labor Statistics 2011). The industry is characterized as a high-stakes endeavor facing uncertainty from many sources (Miller and Lessard 2001) that is tradition-bound, seasonal, capital intensive, and for which each project is unique and physical. The result is schedule delays and cost overruns, the negative effects of the essential dimensions of construction project management: time, cost, and resources (Kerzner 2003). They are interrelated and particularly vulnerable to variability.

To establish the foundation of the negative impacts on construction project network schedule systems, risk and uncertainty are defined in their various forms, beginning with Keynes' (1936) assessment of the importance of risk awareness and Knight's assertion that risk can be measured while uncertainty cannot. Beyond these basic definitions, risk categorization is explored through various taxonomies and its quantification expressed through the ubiquitous Composite Risk Index.

Risk management, the ability and/or opportunity to assume, not assume, abate, allocate or transfer risk (Abramowitz 2009) translated into the Jaafari (1984) and Glavinich (1994) objectives for project planning, whose foremost elements considered by this research are the balancing of uncertainty and modification and the ability to allocate responsibility. Project schedules are the vehicle to implement planning objectives and manage risk, although the expectation for schedule overruns at the outset of a project is generally not viewed as probable (Ambani 2004).

The critical path method of scheduling facilitates the discrete scheduling of time and activities to an acceptable level of accuracy and is ubiquitous to construction projects (Galloway 2006). It began with Kelley and Walker (1989, 1959) in the middle part of the 20th Century and makes possible the realization of a multiplicity of float types (the slack time, lag, or buffer that the start of an activity may be delayed) based upon solving a series of linear equations of the "*start plus duration equals finish*" form.

Total float, the time in which an activity can be delayed before impacting the overall project / schedule network is recognized as a key issue in project management (de la Garza et al. 1991), as its consumption does not impact the ultimate completion of the project. The ownership of total float and the ability to trade float, the kernel of the de la Garza, Vorster, and Parvin (1991) seminal paper *Total Float Traded as Commodity*, forms the basis of this research.

> Because both owners and contractors can gain or lose if unforeseen conditions effect the project scope or project schedule, contractors not only have the right to administer and use total float but also the obligation to trade it. Thus, to maintain equilibrium from agreed-on-risk-sharing expectations, flexible time taken away from the schedules needs to be replaced with monetary contingencies (de la Garza et al. 1991, p.719).

Float is recognized as a *highly valuable* but *diminishing* commodity (Wickwire et al. 2003) that is governed by contract and tradition. It belongs to no specific entity (owner, contractor, subcontractor, etc.) and is consumed on a first-come, first-served basis (Pasiphol 1994), that results in problems when float entitlement is asserted by more than one entity. This behavior links the still-unanswered question, *"Who owns float?"* (Person 1991) with the equally important *"Who should own risk?"* Multiple float ownership or allocation precepts have arisen, including owner entitlement, contractor entitlement, and joint ownership (project based control predicated upon a distributive mechanism or set of determining factors), with joint / project ownership being the focus of this research.

Irrespective of entitlement, float is consumed for the benefit of project participants to overcome the failure to make timely decisions, accommodate changes in the scope of work, mitigate interference with the work of others, coordination of the work is lacking, when there is a failure to perform respective duties and responsibilities, when the schedule is inadequate and in flux, or through inaction of sub-entities (Hulett 1995, Vezina 1991, Householder and Rutland 1990).

The allocation of float to individual activities, under scheduling convention, by contractual agreement, mere consumption on a first-served basis, or by trading as posited by de la Garza et al., can take several forms: uniform distribution, distribution based on activity duration, or distribution based on activity cost (Pasiphol and Popescu 1995, Pasiphol 1994).

Several alternative theories to the management of projects and their schedules exist. Beyond CPM, and the soft logic methods evaluation methods of Graphical (GERT), Venture (VERT), and Program (PERT) Evaluation and Review Techniques respectively, two other methods for optimizing schedule performance have advanced: Critical Chain Project Management (CCPM) developed by Goldratt (1997) originates in the *Theory of Constraints*; and Lean Construction philosophy, an extension of lean manufacturing principles and practices stemming from the Toyota Production System. These methods are applied to the construction process with a concern for continuous improvement and the abandonment of the time-cost-quality tradeoff paradigm.

CCPM seeks to aggregate all of the 'safety time' that is added to the tasks within a project into a single quantity (typically aggregated into a single buffer at the end of the project) so that it is not wasted through poor performance across the project. Conversely, Lean Construction is not schedule or duration focused, but rather seeks to eliminate waste, organize production efficiently and reliably, and deliver a product meeting expectations and/or customer needs absent any inventory. It is concerned with the holistic pursuit of concurrent and continuous improvements in all dimensions of the built and natural environment (Abdelhamid 2007).

To fulfill the research objectives, process-centric modeling (discrete event simulation) was employed to establish a mathematical model through Monte Carlo simulation to validate the hypothetical suppositions relative to allocating float (risk), valuing (price) float, and correlating risk among project participants. This required the investigation and selection of appropriate probability distribution functions (PDFs) to approximate the uncertainty and variability experienced by construction activities. The work of Wilson et al. (1982), Touran (1997), Back et al. (2000), Fente et al. (2000), Schexnayder et al. (2005), and Arízaga (2007) serves as a guide in determining the appropriateness of specific PDFs.

The background portion of Chapter One concludes with identification of the primary purpose of the research endeavor: to seek proven concepts from the areas of finance and social decision-making and through extension and adaptation transfer this knowledge to network scheduling systems to show how systematic risk can be quantified, priced, diversified and/or mitigated using accepted and seemingly unrelated concepts in fulfillment of the "Total Float Traded as Commodity" notion posited by de la Garza, Vorster, and Parvin (1991) 20 years ago. It is based upon the in-depth literature review that *no approach exists that provides a theoretical framework for the fair quantification, allocation and valuation of float to mitigate risk in projects governed by network schedule systems*.

5.1.2 Portfolio Theory Analogy (Chapter 2)

The portfolio theory section of this dissertation begins with the supposition that an as-planned schedule is analogous to perfect competition as set forth by Adam Smith (1723 – 1790). Perfect competition occurs in a market where any entity may enter and leave as they desire and where no single entity holds power over the entire market. By extension to network schedule systems, the as-planned schedule is considered 'perfect,' as no single activity or participant may be large enough to have the power to dictate and control the completion of the project. Perfect market competition, like perfect schedules (ones withg no deviations) rarely exist. However, the interaction among market and/or network schedule participants becomes of interest to this research.

To understand the relationship and interaction among market participants, the origins of contemporary financial markets and common terminology, are defined. This includes the difference between generic financial markets, i.e. the stock market aimed at bringing together buyers and sellers to complete a transaction, versus functional markets, i.e. exchanges where specialized intermediaries transact under a common set of rules in a closed system. Functional markets are established with specific intent, specialization relative to stocks, bonds, futures contracts, etc. (Michie 1999). The purpose of the market is presented as twofold: price discovery and financial liquidity.

The origin of the financial market is traced as far back ancient Mesopotamia and the clay tablets recording interest-bearing loans. Exchangeable bonds came into being during the Italian medieval and Renaissance periods surrounding agriculture and spice trade shipping, with the earliest organized market forming during the 12th and 13th centuries. A large portion of early financial markets surrounded the financing of war efforts by governmental treasuries, particularly in Venice. Venetian debt trade through their war bonds spanned centuries and spawned individual trading houses, like the *Famiglia de' Medici* or House of Medici (the first bank to trade debt across an entire country).

During the 17th century, the Dutch East India Company (or VOC) became the first publically traded company, and for which the oldest remaining stock certificate exists. The leadership of the VOC first recognized that trading stock could be a profitable endeavor. "'This little game [the trading of VOC shares and derivatives] could bring in more money than contracting charter parties for ships bound for England,' wrote Rodrigo Dias Henriques to Manuel Levy Duarte on 1 November 1691" (Petram 2011, p.1).

The ascension of Dutch ruler William of Orange (1650 – 1702) to the throne of England, Scotland, and Ireland after the Glorious Revolution of 1688 brought organized financial markets to England and ultimately to the United States. Transactions in England and the U.S. initially took place in the street, in back alleys, and in coffeehouses. The Buttonwood Agreement (1792) set the stage for organized trading in the US, which would lead to the New York Stock Exchange (NYSE), the American Stock Exchange, and ultimately to electronic trading on the NASDAQ.

Stock market behavior, the element of interest to this research, includes concepts such as Groupthink, the Efficient Market Hypothesis, and other forms of irrational behavior, with the most notable and colloquial (a 'genericized' trademark, a proprietary eponym) being the Greenspan *irrational exuberance* characterization of the late 20th and early 21st century stock market performance (Shiller 2005).

Emanating from the study of stock market behavior and/or performance is the 1990 Nobel Prize concept of the Capital Asset Pricing Model (CAPM). Introduced by Treynor (1961, 1962), Sharpe (1964), Lintner (1965), and Mossin (1966), building on the earlier work of Markowitz (1952), and characterized as "one of the most important advances in financial economics" (Ross et al. 2005, p.295), CAPM presents a method to determine the theoretically appropriate required rate of return of an asset, typically a stock, when the stock is to be added to an already well-diversified portfolio. Of particular relevance to this research is the beta (β) component of the CAPM formula, the correlation of the performance of an individual asset to that of the entire market, where,

$$\beta_i = \frac{\text{Cov}(R_i, R_m)}{\text{Var}(R_m)} \quad \text{[Eq. 5.1]}$$

and

R_i = the return of an individual asset

R_m = the return of the overall market

β_i = beta is the sensitivity of the expected excess asset returns to the expected excess market returns

Within market finance, beta is the number describing the relation of an asset or portfolio's relation of its returns with those of the overall financial market (the S&P 500 Index). It is also a measure of financial elasticity, relative volatility, diversifiable and systematic risk, and liquidity. In simple terms, beta is "the influence of the overall market's return on an individual stock" (Smith 2003b p.176), the asset-specific historic coefficient representing the degree to which an individual stock moves with the market.

Nested within the CAPM is risk, specific or unsystematic risk (that which can be diversified, mitigated or eliminated) and systematic or market risk that cannot be eliminated by diversification (e.g. equity, interest rate, currency, and/or commodity risks). The CAPM, by default, becomes the means to measure systematic risk (McClure 2010).

Network schedule systems surrounding construction projects participate in a complex decision-making process subject to constraints and uncertainties as to whether a project will be on time. Where individual assets comprise a market, many activities and entities compose a project. The CAPM is an analogous determinant for the behavior of those participating in construction projects and a measure of a risk within network schedules.

Float is the measure of flexibility that reduces risk and increases opportunity within construction projects (Thompson and Lucko 2011). It quantifies the ability for an entirety of schedule participants to accommodate uncertainty and absorb delays. The CAPM becomes *apropos* to schedule systems with beta being the measure of interaction within the systems and a measure of the need for flexibility. In its ultimate form, it becomes the measure of risk leading to the expenditure of float. The beta component of the CAPM can be extended to the performance of an individual activity / entity within a project network schedule system and ultimately to a collection of construction projects.

5.1.3 Voting Model Analogy (Chapter 3)

The voting model section of this dissertation begins with a discussion of risk and its various definitions, applications, and taxonomies. Much has been written about the cause, effect, mitigation, avoidance, and transfer or shifting of risk, but there remains no standard practice for its valuation or allocation. Risk is simply defined as the effect of uncertainty on objectives, whether positive or negative (ISO 2009), and specifically as the probability or threat of a damage, injury, liability, loss, or other negative occurrence caused by external or internal vulnerabilities, that may be neutralized through pre-mediated action (Webfinance 2010a). Beyond weather conditions, project-related mishaps, or other external influences, risk is most recognizable and quantifiable by its impact on project schedules as measured by consumption of float.

Risk related to construction projects and schedule systems can be categorized broadly as institutional, market and completion risks (Miller and Lessard 2001), or narrowly construed as political, social, natural, financial, economic, commercial, technical, logistical, and construction risks (Baloi and Price 2003). Risk related to construction activities and network schedule systems can be addressed through four primary strategies: transference, avoidance, mitigation, and/or acceptance. Risk management is the process of determining the maximum acceptable level of overall risk and through risk assessment techniques to determine if it is excessive, and then developing a strategy to reduce it to an acceptable level (Webfinance 2010b).

The unifying factor among all risk classifications is the potential to impact a construction project schedule, herein characterized as schedule risk: the need for a project (schedule) to exhibit flexibility to absorb externally-influenced and/or externally-originated delays. It is inextricably linked to float, which can be impacted in two ways: positively (where the time required to complete individual tasks is less than the schedule identifies, causing the schedule to advance or finish quicker), or negatively (where task durations are exceeded, and schedule delays can be expected). Seven primary factors cause construction project delays, all of which can be viewed through and impact a network schedule system: owner interference, inadequate contractor experience, financing and slow payments, labor productivity, slow decision-making, improper planning, and subcontractors (Odeh and Battaineh 2002).

The has been considerable speculation, research and mathematical modeling surrounding the way in which entities combine their preferences, needs, and choices into a decision to overcome a perceived risk. There are essentially two methods to make such choices or decisions (Arrow 1964): voting and the inherent mechanisms of the marketplace. Larger groups tend towards voting and elections, and smaller groups favor committee structures and consensus (Lieberman 1971). The determining process to undertake decisions-making in the social context has six elements (Lieberman 1971): (1) distribution of power, (2) joint welfare function, (3) bargaining and coalition processes, (4) differences and characteristics of the participants, (5) group processes or phenomena, and (5) previous experiences and commitments of the members and the possibility of future interaction.

The preponderance of these elements can be found in the decision to expend float, with the distribution of power, or the lack thereof, being of foremost concern. A common processes by which social choice is made, the manner in which the collective integrates individual preference into a decision, is by voting (Birnberg and Pondy 1971), for which the rules and the exercise of control through voting power are complex.

The work of Lionel Penrose is recognized as the earliest scientific examination of the measurement of voting power. Penrose proposed a probabilistic measurement of hypothetical votes in the newly formed United Nations General Assembly, arguing that the equitable distribution of voting power in the assembly should be proportional to the square root of the population represented. His was concern was that it was impossible to create a voting scenario in which every human being would have equal power unless the assembly is formed of nations of equal size. Absent this, "…if large and small nations have equal voting powers, the spokesman for small nations are felt to be too significant, and artificial rules about the meanings of votes and vetoes have to be constructed to redress the balance" (Penrose 1946).

Penrose concluded that the *a priori* voting power of each representative in the voting body should behave proportionally to $\sqrt{n}$, eponymously known as the Penrose Square Root

Law. Further mathematical modeling surrounding voting came into being parallel to and because of the emergence of game theory, which serves as the vehicle for the study and prediction of varying types of systems (with voting models being a subset thereof). Game theory is focused on human processes in which the individual decision-making entities are not in complete control of their respective outcomes.

Disparate entities (general contractors, subcontractors, etc.) participate in a complex decision-making process subject to constraints and uncertainties as to whether the project will be completed on time. Where votes comprise an election, many risks and uncertainties compose a project. Voting theory typifies a possible determinant for the behavior of those participating in large construction projects. The voting population size of the different countries is analogous to the duration or monetary value of the different activities by which the network schedule systems is formed. As a vote is the vehicle by which decisions are made, the decision to expend float is the analogous vote within the schedule system. The only difference between elections and projects is that project activities occur within a dependency structure, whereas blocs cast their votes all at the same time.

Both elections and projects are modeled as binary decision-making processes. Voters have a quantifiable influence (their voting power) on the decision, so, too, do subcontractors have influence (their performance) on whether the project will be on schedule and within budget.

5.1.4 Real Options and Decision Irreversibility Analogy (Chapter 4)

The real options section of this dissertation begins with the 1983 recognition by the future chairman of the U.S. Federal Reserve, Ben Bernanke that the presence of uncertainty can increase the value of delaying decisions (Bernanke 1983). Subject to two simple assumptions, decision irreversibility, and the availability new information, Bernanke concluded that postponing a decision, while maintaining the ability to commit at a later time, can prove desirable by allowing choice only after important information is revealed. He maintained that the option to make a decision in the future has inherent value. This conclusion extends credibility to the primary thesis of options theory and in particular real options in decision-making and valuing flexibility.

Option theory, and in particular real options (those dealing with tangible assets), existed as early as ancient Greece (the Thales of Miletus olive press rights) and is even referenced in the writings of Shakespeare. Options theory, as well as the Bernanke supposition, is predicated upon the existence of risk, as found in uncertainty and volatility. Due to the acceptance of these conditions within the market (and within construction project schedule systems), the meaning of uncertainty and risk have become interchangeable in the contemporary lexicon.

Knight (1921, p.19) first distinguished between measurable risk and "unmeasurable [sic] uncertainty." Strangert (1977, p.35) interprets Knight as "uncertainty refers to an unstructured perception of uncertainty and risk to the situation in which alternative outcomes have been specified and probabilities been assigned to them." Merrill and Wood (1991) observed that uncertainty refers to factors neither under control nor known with certainty, whereas *risk* is a hazard because of *uncertainty*. Beyond risk and uncertainty, several other terms enter the mix: flexibility, the potential or the capability to respond to change (Mandelbaum and Buzacott 1990, Slack 1983) (the operationalization of the Bernanke supposition and manifestation of waiting for better information), and robustness in the context of flexibility and uncertainty represents the ability to satisfactorily endure all envisioned risks or contingencies.

"The value of flexibility is a function of variation in price and how well that variation can be predicted before the decision is made" (Marschak and Nelson 1962, p.52). It is a function of the specific uncertainty with which flexibility deals and the quality of information regarding the uncertainty. Based on this relationship, the greater the uncertainty, the greater the value of flexibility (Ku 1995).

Financial markets have learned how to manage both the positive and negative sides of uncertainty by using hedging instruments such as futures and options. The ability to manage uncertainty is a derivative of the capacity to hedge risks and can be found in its general form by the introduction of flexibility into project designs (Ramírez 2002). It is treated by extension in this research as the ability to manage risk and uncertainty within network schedule systems by the consumption of float.

"The capacity of uncertainty to be resolved in the future is precisely the characteristic that allows it to generate value" (Ramírez 2002, p.24). This represents the merger of uncertainty and risk, with real options, and the Bernanke concept of value in delaying decisions.

The capital budgeting component of corporate finance deals with the monetary decisions that businesses make and has developed a variety of tools and analysis methods to guide the process and lend insight into the long-term versus short-term benefits and choices. Short-term decisions focus on the 'here and now' operation of projects and become *apropos* to inherent risk and the decisions to consume float and are germane to construction projects governed by network schedule systems.

The analytical methods used in capital budgeting decisions fit into two categories (Ross et al. 2005): discounted cash flow methods and non-discounted cash flow methods, with the primary difference being the Time Value of Money principle.

Table 5.1: Capital Budgeting Analysis Method Categorization

Cash Flow Method	
Discounted Type	**Non-Discounted Type**
Net Present Value	Payback Period Method
Cost-benefit Ratio	Return on Investment
Profitability Index	Accounting Rate of Return on Investment
Internal Rate of Return	
Modified Internal Rate of Return	

There are challenges associated with use of these analytical methods. Non-discounted cash flow methods ignore the time value of money altogether, while the majority of those recognizing it (discounted cash flow methods) foster uniformity with subjectivity in the application of discount / interest rates. Beyond the shortcomings and subjectivity associated with specific idiosyncrasies, traditional capital budgeting methods have difficulty in valuing project abandonment, deferral, or alteration in any way (Mbuthia 2001). They cannot adequately recognize managerial operating flexibility and strategic interactions. The end result is an "inability to properly recognize the value of active management in adapting to changing market conditions" (Trigeorgis 1996, p.9).

A financial option is generically defined as "…a security giving the right to buy or sell an asset, subject to certain conditions, within a specific period of time" (Black and Scholes 1973, p.637) and was first traded on an exchange in 1973 (Hull 2000). Financial options are derivative instruments based on an underlying asset (generally stocks, bonds, currency, commodity futures, etc.) that form a contract between parties to buy (a call option) or sell (a put option) the underlying asset at a predetermined fixed price (the exercise price or strike price) on or before a specified expiration or exercise date (Black and Sholes 1973).

Option value may determined by one of several mathematical models, with the expectation of uniform results, but practically returning different values. They attempt to predict how the option value changes in response to the changing conditions in which it exists, and all consider five key elements (Frayer and Uludere 2001): the value of the asset being optioned, the exercise or strike price, the time to expiration (or to the evaluation / decision point), volatility of the asset in its market or setting, and the risk-free interest rate. Option model typologies include: (1) the finite difference method (of which the Black-Scholes Equation is a member) uses partial differential equations to determine a value: (2) Monte Carlo simulation, which produces thousands of possible scenarios for the uncertain elements surrounding the option and generates random price paths that result in an option payoff (value): and (3) the binomial valuation method that uses decision trees and decision lattices to depict the various decisions points (nodes) at which the decision-maker faces a choice that can be valued. Decision lattices are preferred over decision trees due to their recombining nature. This reduces the possible terminal node quantities, as trees increase exponentially while lattices increase by a single node per time period.

Black and Scholes (1973) derived a formula (the Black-Scholes Equation, Eq. 5.2) to determine the value of an option, for which Robert Merton and Myron Scholes were awarded the 1997 Nobel Prize in Economics.

$$C(S,t) = N(d_1)S - N(d_2)Ke^{-rT} \qquad \text{[Eq. 5.2]}$$

where

N(y) = probability, the standard normal distribution
K = strike price of the option
T = time to maturity
S = current stock price (the underlying asset)
r = compounded annual risk-free rate
σ = volatility of the returns of the underlying asset
d_1 = the factor by which the present value of contingent receipt of the stock exceeds the current stock price
d_2 = the risk-adjusted probability that the option will be exercised

The term *real options* was first used by Myers in 1984 in the context of strategic corporate planning and capital budgeting. Real options are founded upon the same principles as financial options, but give the owner the right, but not the obligation, to take future action with respect to real, tangible assets, depending on how uncertain conditions evolve (Amram and Kulatilaka 1999). It is presupposed that when considering real options, real tangible assets are actually owned (Frayer and Uldere 2001). The underlying premise of real options is that wherever there is a choice, there is the ability to benefit from the upside, while avoiding or postponing any downside risk.

There are two distinct differences between financial and real options. The information necessary to value a financial option and ultimately make the investment decision is readily available to all interest parties. The information surrounding real options is generally proprietary and unavailable to all but direct participants in the decision process (Copeland and Tufano 2004). Secondly, real option terms are often less clear that their financial counterpart. The ability to exercise a financial option is prescriptive, and the fulfillment of its transaction is executable in readily accessible markets.

The taxonomy of real option puts and calls characterized by decision action are: (1) the option to defer, (2) the option to abandon, (3) the option to expand, (4) the option to alter, (5) the option to switch, and (6) the option to time or stage. Similarly, real options may also be classified according to their relationship to the specific project: real options 'on' versus 'in' projects or systems (Wang and de Neufville 2006). Real options 'on' projects and systems are similar to call options, as they represent the right, not the obligation, to invest in a project. Real options 'in' a system are far more complex and difficult to identify. Their application considers the inner workings of the project or system. Real options 'in' a project originate from the system's design and entail an appropriate level of internal knowledge.

The methods to value real options are founded upon those for financial options, and there are several as well, including the market approach, the income approach, and the cost approach (Mun 2006). As with financial valuation methods, there are challenges or difficulties associated with real option valuation methods. These challenges surround net present value determination, agency theory (violations of the trust and responsibilities placed in individuals to lead/manage an entity primarily owned by others), overconfidence and the illusion of control, and portfolio pitfalls (the aggregate of multiple small losses).

Float is a measure of flexibility that reduces risk and increases opportunity (Thompson and Lucko 2011). Its various types quantify the ability of an entire schedule or individual activity to accommodate uncertainty and absorb delays. Real options become *apropos* to schedule systems with float being the measure of flexibility within the systems and uncertainty being the driver of and necessitating flexibility. The Bernanke concept of waiting for additional information is appropriate to the multiplicity of activities and the uncertainties surrounding their completion.

Construction project planning efforts include the identification, assessment, and selection of alternative strategies (Ford et al. 2002), which includes the management of uncertainty throughout the planning and construction periods. Therefore, "[t]he flexible decision structure considered in option theory is also valid in scheduling . . . a real options analysis should be used" (Boute et al. 2004, p.2).

5.2 Conclusions and Findings

The purpose of this research is to apply proven concepts from the areas of finance and social decision-making and by extension and adaptation transfer this knowledge to network scheduling systems to show how systematic risk can be quantified, priced, diversified and/or mitigated using accepted but seemingly unrelated concepts. The research is an extension of *Total Float Traded as Commodity* notion from by de la Garza, Vorster, and Parvin (1991).

This remains unrequited in its entirety despite far-reaching conversations, proposals, and hypotheses in the literature.

> [W]hether total float is perceived as a time contingency or as an incentive, its potential opportunity value is neutralized when consumed by owners. A revised model for pricing total float is needed. Such a model should allow for the trading of total float by making its commercial opportunity value explicit. The revised model should grant the contractor the right to administer total float, impose on the contractor the obligation to disclose its value and trade it on demand (de la Garza et al. 1991, p.719).

The mechanisms for trading total float rests with the research activities like this:

> There is clearly a need to examine issues such as the trade-in value of float and many others relating to the practical implementation of CPM technology for managing construction …These issues must be addressed by academics who provide the [necessary] intellectual leadership… (de la Garza et al. 1991, p.727)

This research effort recognizes that the body of knowledge lacks a comprehensive examination of the fundamental issues surrounding trading total float as a commodity. The absence of the measurement of where risk resides (risk being the precursor to the consumption of float), the distribution of total float for consumption, and at what price, engenders the who, what, where, when, and how of *Risk Measurement, Allocation, and Pricing in Network Schedule Systems*. In fulfillment of this objective, a comprehensive total float management strategy is presented in three parts.

5.2.1 The Location of Risk in Network Schedule Systems: Research Objective 1 – Chapter 2

The conclusions reached with respect to risk measurement and identification required investigation of existing methodologies for the identification of risk and quantification of its presence within network schedule systems. To accomplish this, three questions were posed at the beginning of this research and are answered herein.

Question 1: What proven methodologies exist for determining where risk resides within an organizational structure?

Question 2: What proven methodologies exist for determining where risk resides within or among a group of related unstructured entities operating in relation to each other?

Question 3: Can these methods be adapted or translated to network schedule systems to bring meaning and/or clarity to risk identification and measurement for a meaningful purpose?

Risk has been shown to impact a network schedule system from two perspectives, within the system and external to the system. External risks are not considered in this portion of the research; a the focus is on the internal relationships of the network participants. Financial portfolio theory was examined as an allegory to the internal interaction of schedule participants and that of the overall network schedule system. The results in answer to the initial questions are summarized in Table 5.2.

Table 5.2: Research Objective Summary – Portfolio Theory Analogy

Research Objectives		
Research Question	**Answer / Contribution**	**Location**
1. Proven methodology for risk within organizational structure	Financial Portfolio Theory	Table 2.4
2. Proven methodology for risk among a group of related / unequal entities	The Capital Asset Pricing Model	Section 2.2.5 Section 2.3.1
3. Can methodologies be adapted / extended to network schedule systems with meaning	The CAPM beta coefficient can be used to describe the risk relationship between schedule participants and the overall system	Section 2.4.3.1

The elements of financial portfolio theory and the Capital Asset Pricing Model introduced by Treynor (1961, 1962), Sharpe (1964), Lintner (1965), Mossin (1966), and Markowitz (1952), fulfills the criteria of an existing theory and/or accepted methodology that is proven (recognition by the committee for the *Alfred Nobel Memorial Prize in Economic Sciences* in 1990 is considered sufficient). The CAPM includes a measure to describe the risk (the expected behavior) of related but unequal entities residing in a common system, the beta coefficient (Eq. 5.1).

It has been shown that a financial market, whose collective activities encompass multiple exchanges, investors, and individual assets, is analogous to a construction project's participants as defined by a network schedule system. Elements of financial portfolio theory are translated to reflect construction project management and network schedule systems as highlighted in Table 5.3.

Table 5.3: Summary of Comparison of Portfolio Theory and Project Management

Portfolio Theory Element	Project Management Correlation
Financial Market	Construction Industry
Specific Exchange	Construction Project
Portfolio	Collection of Activities / Subcontractors
Weight	Value or Duration of Activity
Asset *(Stocks)*	Subcontractors *(Different specialties / trades)*
Market Transaction	Project Execution
Transaction Results	Activity / Subcontractor Performance
Market Performance	Schedule Performance
Market Mover	Critical Activity
Systematic Risk	Project Risk
Specific Risk	Activity Risk

Similar to the analogies depicted in Table 5.3, network schedule systems have components that are directly analogous to those required for a CAPM beta calculation. They are summarized in Table 5.4.

Table 5.4: Summary of Network Schedule System Analogous Beta Components

CAPM Beta Input	Schedule System Beta Input	Description	Variable
Asset Rate of Return	Activity Performance	Activity Actual Performance	P_a
Asset Return Benchmark	Activity Performance Benchmark	Activity As-Planned Duration	$\overline{P_a}$
Market Rate of Return	Project Cohort Performance	Project Actual Performance	P_c
Market Return Benchmark	Project Cohort Performance Benchmark	Project-Level As-Planned Durations (for each Project Forming the Cohort)	$\overline{P_c}$
Financial Beta Significance	Schedule Beta Significance	Relationship of Activity Performance (Duration) to that of the Cohort	β_c

By depiction through an exemplar and validation by way of Monte Carlo simulation, beta (β) has been shown to be an appropriate measure of interaction within the systems and a measure of the need for flexibility and the expenditure of float. The calculation of beta within a network schedule system has been demonstrated to fit within an expected range, conforming to the theoretical constraints enumerated below:

Theoretical Range for β_c: $-1.0 < \beta_c < 1.0$

Expected / Practical Range for β_c: $-0.25 < \beta_c < 0.50$

$\left(\left|P_a - \overline{P_a}\right|\right) \ngtr P_c$ and $P_a \ngtr P_c$

except that $\left(P_a - \overline{P_a}\right)$ may be $> (P_c - \overline{P_c})$

Similar to the meaning and stratification of financial betas, networks schedule system beta calculation results (obtained through exemplar schedule system simulation) can be stratified and lend insight to activity performance. The conclusions regarding construction schedule system activity betas (β_c as defined by this research) are summarized in Table 5.5.

Table 5.5: Summary of Schedule Risk Classification by Beta Range

Beta (β_c)	Risk Insight	Characteristic / Meaning
$\beta_c > 1$	N/A	Not possible – activity schedule duration cannot exceed that of project
$\beta_c = 1$	Extreme Risk	Responsible for ALL schedule delays within the cohort of projects considered
$0.5 < \beta_c < 1$	High Risk	Responsible for the majority of schedule delays experienced on projects
$0.25 < \beta_c < 0.5$	Moderate Risk	Performance in excess of that expected with typical project risks
$0 < \beta_c < 0.25$	Low Risk	Performance within acceptable limits and generally accepted project risks
$\beta_c < 0$	Safe or Anti-risk	Performs better than as-planned schedule durations

Simulation results also depict and this research concludes that activity betas (β_c) mimic the supply chain (operations research) phenomenon known as the 'bullwhip effect,' as originally defined by Forrester (1961). Absent external influences and periodic schedule updating, the magnitude of activity betas increases with relative schedule position (larger during later schedule segments) and with relative activity duration length.

This conclusion of this dissertation research segment is that the specific risk exhibited by schedule system participants can be measured by the CAPM beta component. Further, it has been shown that network schedule system activity betas (β_c) represent the difference between aggregate historic activity performance and the perfectly competitive schedule across multiple projects.

5.2.2 The Allocation of Risk in Network Schedule Systems: Research Objective 2 – Chapter 3

The conclusions reached with respect to risk allocation required investigation of existing methodologies for the identification of fair allocation, apportionment and/or distribution methods. To accomplish this, four questions were posed at the beginning of this research and are answered herein.

Question 1: What proven methodologies exist for allocating or apportioning something across an organizational structure?

Question 2: What proven methodologies exist for allocating or apportioning something across a group of related unstructured entities operating in relation to each other?

Question 3: What established processes are used and/or factors exist for determining/reaching a decision regarding something within an organizational structure among a group of related unstructured entities?

Question 4: Can these methods or processes be adapted or translated to network schedule systems to bring meaning and/or clarity to risk identification and measurement for a meaningful purpose?

Float is the measure of flexibility that reduces risk and increases opportunity. It quantifies the ability of an entire schedule or individual activities therein to absorb delays. However, float is not available to all participants in a network schedule system. By definition critical activities have no float, and consequently, no flexibility to absorb project risk generated delays. Coupled with the recognition that contractors reserve time at the end of a project to absorb delays (defined by this research as contract float – the difference between network schedule duration and that required by the contract), this research presents a method to allocate flexibility to critical activities. The results in answer to the initial questions are summarized in Table 5.6.

Table 5.6: Research Objective Summary – Voting Model Analogy

Research Objectives		
Research Question	**Answer / Contribution**	**Location**
1. Proven methodology for allocations across an organizational structure	Social Choice	Section 3.2.4
2. Proven methodology for allocation across a group of related / unequal entities	Penrose Square Root Law	Section 3.2.5.1 Section 3.3.1
3. Established processes for decision-making by an organization / group	Voting Models	Table 3.3
4. Can methodologies be adapted / extended to network schedule systems with meaning	The Penrose Square Root Law can be used determine the proportional allocation of contract float (CF) each critical activity receives (distributed float (DF) as defined by this research)	Section 3.4.4

The elements of social choice and the Penrose voting model fulfill the criteria of an existing theory and/or accepted methodology that is proven (half a century of a voting allocation eponymously described is considered sufficient). The Penrose Square Root Law provides a method to fairly apportion contract float across critical schedule activities, those that by definition have no float.

It has been shown that groups make decisions in two manners, either through voting or by market mechanisms and that the decisions made by construction project network schedule system participants (the subcontractors owning the array of schedule activities), are analogous to casting votes. The collection of votes (decisions) across the network schedule represents an election. This enables the Penrose Square Root Law to be translated to construction project management and network schedule systems per Table 5.7.

Table 5.7: Summary of Network Schedule Systems Analogous Decision-Making and Political Voting Components

Voting Elements	Project Management Correlation
Federal System	Schedule network
Representation	Non-self-performing, outsourcing
Election	Project
Campaigning	Planning
Voters	Subcontractors
Vote	Decision to Expend Float
Ballot (Voting process)	Schedule and budget (Project controls)
Voting Power	Influence on Performance
Swing vote	Critical participant

Building upon the analogies depicted in Table 5.7, network schedule systems have two attributes that are appropriate measures to serve as the base to which the Penrose Square Root Law is applied (the statistic by which float is to be allocated by square root proportion to critical activities). Using an exemplar and validation through Monte Carlo simulation, activity duration is shown to be the appropriate attribute by which to allocate float (herein defined as distributed float (DF)) to critical activities.

To distribute float via the square root proportion of activity duration, normalization of the calculation results is due to the critical activities representing partial and/or incomplete duration arrays (aggregate durations that do not represent 100% of the network schedule completion time). Similarly, integer rounding is required to allocate float in whole day increments (partial days are anathema to scheduling), just as votes are integers per

Sainte Laguë and d'Hondt. The advantages of allocating contract float across critical activities are summarized in Table 5.8.

Table 5.8: Distributed Float Contribution to Contemporary Float Attributes

Float Characteristic	**Decision-Making Allocation Model Application**
Diminishing Commodity	Distributed float (DF) diminishes as critical activities conclude (Figure 4).
No Direct Ownership or Control	The allocation model distributes contract float (CF) and fosters ownership by those most in need – critical activities
First Come, First Served Consumption	By allocating contract float (CF) as distributed float (DF) thereby instituting an ownership atmosphere, FC-FS consumption is negated in favor of consumption directed by time and 'controlling' critical activity

This conclusion of this dissertation research segment is that flexibility can be brought (DF allocated) to critical activities by application of the Penrose Square Root Law. Further, it is concluded that the duration square root allocation model is superior to that of the nominal duration (whole duration-based, not a square root derivative) and superior to that of both cost-based models (nominal and square root).

5.2.3 The Pricing of Risk in Network Schedule Systems: Research Objective 3 – Chapter 4

The conclusions reached with respect to risk allocation required investigation of existing methodologies for the identification of fair allocation, apportionment and/or distribution methods. To accomplish this, three questions were posed at the beginning of this research and are answered herein.

Question 1: What proven quantification/pricing methodologies exist to establish the price of risk within an organizational structure?

Question 2: What proven quantification/pricing methodologies exist to establish the price of risk among a group of related unstructured entities operating in relation to each other?

Question 3: Can these methods be adapted or translated to network schedule systems to accurately quantify/price risk in a meaning way?

Risk can manifest and impact projects governed by network schedule systems at any time. The ability to be flexible and absorb delays may or may not be present in the form of float. Consequently, the ability to delay a decision regarding risk is valuable. The real option component of capital budgeting and finance was examined as an analogous method to price float for trade across the network system critical activities. The answers to the initial questions are summarized in Table 5.9.

Table 5.9: Research Objective Summary – Real Option Analogy

Research Objectives		
Research Question	**Answer / Contribution**	**Location**
1. Proven pricing methodology for risk within organizational structure	Capital Budgeting and Finance	Table 4.10
2. Proven pricing methodology for risk among a group of related / unequal entities	Real Options	Section 4.2.4.12 Section 4.3.3
3. Can methodologies be adapted / extended to network schedule systems with meaning	The binomial valuation method of real options theory can be used to value float for exchange (by extension price risk) among critical schedule participants	Section 4.4.3.1

The real option valuation methods of capital budgeting and corporate finance fulfills the criteria of an existing theory and/or accepted methodology that is proven (recognition by the committee for the *Alfred Nobel Memorial Prize in Economic Sciences* in 1997 for the Black-Scholes Equation is considered sufficient). Real option theory includes a method to value the flexibility necessitated by the presence of risk among related but unequal participants, the binomial valuation method.

It has been shown that the flexibility offered by a real option is analogous to the inherent flexibility represented by float within a construction project's network schedule system. The valuation methods emanating from capital budgeting and finance are translated to determine the value of float from two perspectives: an activity specific value at which float should be acquired and a value at which float should be offered for sale. Elements from real options theory are translated to reflect construction project management through network schedule systems as highlighted in Table 5.10.

Table 5.10: Summary of Comparison of Real Options Theory and Project Management

Real Option Input	Float Correlation		Valuation Input Consideration
Present Value of Investment	N/A		Investment Cost
Investment Cost	Daily Overhead / General Conditions Costs		Unique to Each Activity
Time Period	Individual Decision Occurrence (a onetime opportunity)		1
Value of Money Over Time	N/A		0
Volatility / Uncertainty	Buyer Side	Schedule Volatility	Unique to Each Activity
	Seller Side	Probability for Needing Float	Unique to Each Activity

Using an exemplar and validation through Monte Carlo simulation, the binomial valuation method has been shown to be an appropriate pricing method for the flexibility represented by distributed float (DF) and its ultimate expenditure. The float pricing calculation within a network schedule system requires the inclusion of an investment cost. It is concluded that the appropriate value within construction projects governed by network schedule systems is the daily overhead / general conditions costs.

The difference between a late finish and modifying operations to meet the schedule is the extended overhead and/or general conditions costs. This accommodates one side of the real option transaction, the call value (buy side transaction) or need to acquire float. Similarly, when float is available for exchange, the put value (sell side transaction) or ability to forego float by sale, a corresponding investment cost is needed. This research concludes that the same overhead and/or general conditions costs should be employed. It is the foundation by which added scope would be marked up.

This conclusion of this dissertation research segment is that the flexibility represented by float within networks schedule systems can be priced by the extension of real option valuation methods. Further, it is concluded that for a real option approach to price float for exchange, the underlying asset is equal to the daily cost to operate. It must be accompanied by appropriate values for uncertainty; both inherent to the schedule network and to its participants.

5.3 Research Contribution to the Body of Knowledge

The main contribution of this research is the development of a quantitative framework that allows network schedule participants to evaluate the systematic risks presented by the system participants, allocate float across the critical activities (which by definition have no flexibility availability to overcome uncertainty), and price float for exchange (trade) in a market mechanism. Beyond the specific allegories, the contribution of this research is the three-part approach to risk: identification, allocation, and pricing.

5.3.1 Network Schedule System Beta (*β_c*)

This contribution to the body of knowledge derived from the Capital Asset Pricing Model is the translation of the financial beta (*β*) via direct substitution to a form applicable to network schedule systems.

Calculating the beta, the performance of a subcontractor (as represented by an activity within a network schedule system) in relation to the performance of the overall project against the as-planned durations, for the participants within construction projects can be accomplished via direct substitution of the variable concepts summarized in Table 5.4 into Eq. 5.1 resulting in the following:

$$\beta_c = \frac{\mathrm{Cov}\,(P_a, P_c)}{\mathrm{Var}\,(P_c)} \qquad \text{[Eq. 5.3]}$$

Substituting values yields the explicit form for the calculation of beta for the participants of construction projects governed by network schedule systems:

$$\beta_c = \frac{\sum(P_a - \overline{P_a})(P_c - \overline{P_c})}{\sum(P_c - \overline{P_c})^2} \qquad \text{[Eq. 5.4]}$$

where

P_a = activity actual duration

$\overline{P_a}$ = activity as-planned duration

P_c = project cohort (population) actual duration

$\overline{P_c}$ = project cohort (population) as-planned duration

5.3.2 Network Schedule System Square Root Rule

This contribution to the body of knowledge derived from the Penrose Square Root Law is the translation of the square root voting allocation model via direct substitution to a form applicable to network schedule systems.

Calculating distributed float (DF) values for allocation to individual critical activities is accomplished by Equation 5.5

$$\mathrm{DF_i} = \left\| \frac{\sqrt{\mathrm{AD_i}}}{\mathrm{CF}} \bullet \frac{1}{\sum_{x=i}^{j} \frac{\sqrt{\mathrm{AD_i}}}{\mathrm{CF}}} \bullet \mathrm{CF} \right\| \qquad \text{[Eq. 5.5]}$$

which simplifies to Equation 5.6:

$$\mathrm{DF_i} = \left\| \frac{\sqrt{\mathrm{AD_i}}}{\sum_{x=i}^{j} \frac{\sqrt{\mathrm{AD_i}}}{\mathrm{CF}}} \right\| \qquad \text{[Eq. 5.6]}$$

where

AD_i = duration for critical activity *i*

CF = project / schedule system contract float

DF_i = distributed float for critical activity *i*

j = number of critical activities (population)

$\| \;\; \|$ = denotes rounding function to nearest integer

5.3.3 Real Options Valuation Method

This contribution to the body of knowledge derived from the real option binomial valuation method is the translation of the Cox, Ross, and Rubinstein binomial decision tree variant via direct substitution to a form applicable to network schedule systems.

Calculating distributed float (DF) value for exchange (sale or acquisition) by critical activities is accomplished by direct substitution of the variable concepts summarized in Table 5.10 into Equation 5.7, resulting in the following:

$$C(X,T) = (S - K) \bullet u \,/\, (1 - r_f)^{\Delta t} \qquad \text{[Eq. 5.7]}$$

which yields for call and put values, Equations 5.8 and 5.9 respectively

$$C(X) = \frac{K \bullet e^{\sigma_c}}{(1 - r_f)^T} \qquad \text{[Eq. 5.8]}$$

$$P(X) = \frac{K \bullet e^{\sigma_p}}{(1 - r_f)^T} \qquad \text{[Eq. 5.9]}$$

where

C(X) = float call value

P(X) = float put value

K = investment cost

r_f = time value of money (the risk-free interest rate)

T = number of critical activities (population)

Δt = individual decision occurrence

σ_c = schedule volatility (buy side)

σ_p = probability to need float (sell side)

u = volatility of option decision ($e^{\sigma\sqrt{\Delta t}}$)

5.4 Recommendations for Future Research

The research presented in this dissertation contributed to developing the mathematical foundations necessary to measure risk, allocate a means of mitigation, and price risk through the ability to trade float within network schedule systems. The contribution of this research is important because it takes a holistic approach to risk. It indentifies a methodology founded upon extraneous proven concepts from the fields of social choice, decision-making, analytic voting models, financial portfolio theory, capital budgeting, and corporate finance.

This research presents a cohesive look and methodology to fulfill the *Total Float Traded as Commodity* concept of de la Garza, Vorster and Parvin (1991), with specific attention to the segment of a network schedule system has afforded no flexibility, the critical path members. As this is a conceptual methodology posited by exemplar and validated through Monte Carlo simulation, concepts for continued research are recommended. As Olmstead recognized in relation to the seminal element of the research triplet, the methodology to comprehensively address risk within network schedule systems, process remains paramount:

> …[T]he process of performing a dynamic analysis tends to broaden one's view of future possibilities and sharpen the logic of one's thinking about various strategic alternatives. The process itself can be more important than the particular analytic results (Trigeorgis 1995, p.43).

Accordingly, recommendations for future research focus on developing actual data and case studies to confirm the principles presented herein.

5.4.1 Recommendation 1 – Schedule Data

To validate the application of beta (β_c) in network schedule systems, an actual population of schedule data representing a common cohort of activity ownership depicting as-planned schedule durations and actual completed durations should be developed. This database of vetted schedule results will serve as the project population for calculating activity betas.

5.4.2 Recommendation 2 – Banzhalf Power Index

Beyond the application of the Penrose Square Root Law, another voting model analogy may prove fruitful in relation to network schedule systems, namely the Banzhalf Power Index. Where the Penrose Square Root Law provides a method to fairly distribute voting power and herein to allocate float, the Banzhalf Power Index defines the power that an individual voter or block of voters has over the electorate. It warrants similar consideration and investigation to determine if valuable insight into the relationship between schedule activities and/or power exerted one over another can be determined.

5.4.3 Recommendation 3 – Construction Industry Uncertainty / Volatility

A shaping element in an option analysis / valuation effort is the uncertainty factor (σ) employed. No measure of industry, project, or participant uncertainty and volatility exists. Continued research should focus on the quantification of construction industry-specific project-level and activity-level uncertainty and volatility measures, the σ_c and σ_p components of the value equation developed by this research.

5.4.4 Recommendation 4 – Seminal Concept

Research recommendation three identifies the lack of industry-specific uncertainty and volatility measures as a shortcoming and deserving of future research. However, it is proffered that this dissertation has identified an appropriate measure for project and activity uncertainty and volatility. It is recommended that future research undertake an investigation to prove that activity beta (β_c) is an appropriate measure for activity uncertainty, expressed as follows:

$$\sigma_c = \beta_c$$

5.5 References

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APPENDIX A

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